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Deconstructing Dimmitt's Anti-Realism: A New Argument against Church's Thesis

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Abstract
Church's Thesis states that every intuitively computable function is recursive. This ends up being very problematic for the intuitionist. On the one hand, it is a key premise in two prima facie compelling arguments for logical revision. In The Taming of the True, Neil Tennant has recently argued that Dummettian Anti-Realism, Church's Thesis, and the principle of excluded middle together contradict the undecidability of Peano Arithmetic.[1] If Tennant is also right in his assertion that Michael Dummett's own arguments for intuitionist revision are fallacious, then the case for intuitionist revision stands and falls with Church's Thesis. Likewise, formulations of Church's Thesis put forward by intuitionists have long been known to be intuitionistically consistent with the Peano Axioms yet classically inconsisistent with them! While this is perhaps more tendentious than Tennant's argument,[2] one might interpret this result in a similar manner. If Church's Thesis is true, then (since the Peano Axioms are true), classical logic is mistaken. On the other hand, it has also long been known that Church's Thesis entails the incompleteness of intuitionistic logic.[3] In addition, and potentially much more problematic, Church's Thesis undermines the Brouwerian epistemology of mathematics that motivated early intuitionism.[4] It follows from Brouwer's conception of the creative subject that for every set of natural numbers there exists a computation by which one can determine if an arbitrary number is in the set. But, as Kripke pointed out to Kreisel,[5] one can intuitionistically prove that it is not the case that every set of natural numbers is recursive. But then, Church's Thesis entails that Brouwer's creative subject can't exist, as Church's Thesis and Kripke's insight immediately entail that it's not the case that every set of natural numbers is computable.

Keywords: *Deconstructing Dummett's, Anti-Realism*

1.0 INTRODUCTION

One might think that there is a serious problem here. If Tennant is right, then the only correct arguments for intuitionism involve Church's Thesis. Yet Brouwer's conception of the epistemology of mathematics motivating intuitionism is inconsistent with Church's Thesis. As things stand, though, there is only the appearance of a problem, since many contemporary intuitionists such as Tennant do *not* accept Brouwer's neo-Kantian epistemology of mathematics, but instead accept Michael Dummett's neo-positivistic account. Indeed, as noted, it is Dummett's verificationism that Tennant uses in his argument for logical revision.

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[1] Tennant uses the law of excluded middle and Dummett's view that understanding mathematics requires being able to recognize proofs to describe the following decision procedure μ on a discourse D.

> Given any sentence Φ in D, find a speaker who understands it. *Ex hypothesi* we can do this. Set the speaker the recognitional task, with our presentational help, of telling whether Φ is true. (This is the first step of the tandem method.) If the speaker says that Φ is true, record him as delivering the verdict T. If he says that it is not the case that Φ is true, record him as delivering the verdict F. Take the speaker's verdict as the output of μ on Φ. (This is the second step of the tandem method). (Tennant, 1997, 207)

Then Tennnant argues that μ is a computable function. But then by Church's Thesis μ is also a recursive function. Since D can stand for any discourse, including those provably non-recursive (such as Peano Arithmetic), Tennant concludes that the law of excluded middle is false. In (Cogburn, forthcoming) I critically evalute this argument.

[2] For these results, see (Dragalin, 1988), and (Odifreddi, 1996). The problem is that the formal language statements of Church's Thesis under consideration can only fairly be thought of as representations of Church's Thesis if one interprets the quantifiers in the manner of the intuitionists. Consider the following, from (Kreisel 1965).

$$\forall x \exists y \mathrm{R}(x, y) \rightarrow \exists e \forall x \exists z[\mathrm{T1}(e, x, z) \wedge \mathrm{R}(x, \mathrm{U}(z))]$$

Given the intuitionist construal of the quantifiers, any choice function associated with the antecedent will be intuitively computable. This is not the case for the classicist though! So, on the classicist's construal of the quantifiers, this is not a statement of Church's Thesis. So what we really have is that Church's Thesis forces a constructivist to eschew classical logic.

[3] For an overview of results concerning intuitionism and completeness, see (McCarty, 1996).

[4] See (Kreisel, 1970), and (Odifreddi, 1996).

[5] Cited in (Kreisel, 1970).

Pace Brouwerian intuitionism, which takes mathematical objects to be mental constructs of some sort, Dummettians argue for intuitionism by focusing on the *public* nature of linguistic understanding. For Dummett, and contemporary intuitionists such as Prawitz, Tennant, and Wright, it is precisely because of a Wittgensteinian conception of content as *publicly* accessible that they are led to identify grasp of mathematical truths with the (idealized) ability to recognize proofs, and hence to identify mathematical truth with (idealized) provability in the manner of Heyting. Given this, it is of little moment to current intuitionists like Tennant that Church's Thesis undermines Brouwer's creative subject. Indeed, Tennant himself is so sure of Church's Thesis that he labels Stewart Shapiro's questioning of it as "speculative metaphysics." (Tennant, 1997, 208) In the mouth of a somewhat unreconstructed logical positivist like Tennant, this is not a compliment.

Strangely, though, as far back as 1977 Dummett he claimed the thesis to be "not particularly plausible from an intuitionistic standpoint." (Dummett, 1977, 264) The reasoning given by Dummett is enigmatic. The assumption that we can effectively recognize a proof of a given statement of some mathematical theory, say elementary number theory, lies at the basis of all intuitionistic mathematics; but to hold that there is any recursive procedure for recognizing proofs of arithmetical statements would be to run afoul of Gödel's Incompleteness Theorem. (Dummett, 1977, ibid.). Somehow, Dummett takes Gödel's Incompleteness Theorem and the philosophical reasons that motivate intuitionism to undermine Church's Thesis. This raises several questions. First, it is not immediately clear how Gödel's Theorem and Dummett's verificationist epistemology undermine Church's Thesis. Indeed, a reconstruction of the argument Dummett seems to have in mind shows the premise "that we can effectively recognize a proof of a given statement of some mathematical theory" and Gödel's Theorem do *not*, on their own, contradict Church's Thesis. However, the anti-holism and verificationism that (along with Dummett's epistemology) comprise Anti-Realism, *are* enough to yield a valid argument against Church's Thesis. Thus, as I will show, Dummettians must reject Church's Thesis. But then the earlier problem envisioned by the death of Brouwer's creative subject comes back with a vengeance. If Tennant is right, then Dummettian Anti-Realism only entails intuitionism if Church's Thesis is true. But Dummettian Anti-Realism entails that Church's Thesis is false. The second question raised by these issues concerns what to make of Dummettian Anti-Realism without Church's Thesis. After reconstructing Dummett's argument, I will suggest that the denial of Church's Thesis potentially robs Dummett's position of the epistemic virtues associated with it.

2.0 DUMMETT'S ARGUMENT AGAINST CHURCH'S THESIS

Church's Thesis states that there is a procedure to determine whether an arbitrary object is a member of a set if and only if that set is general recursive.[6] Another way to put this is to say that a set is intuitively computable if and only if it is general recursive. An immediate consequence of this is that if there is a procedure to show that an arbitrary member of a set is, in fact, a member of that set (though possibly not a procedure to show that an arbitrary nonmember of the set is not a member) then that set is recursively enumerable. Thus, where "C. T." names Church's Thesis, and "Γ" stands for an arbitrary set, we have the following.

1. C. T. $\vdash$ (Γ is effectively enumerable) $\rightarrow$ (Γ is recursively enumerable)

Craig showed that if a set of sentences is recursively enumerable, then it is axiomatizable. Where "C. R." names this result, we can give the premises in this manner.

2. C. R. $\vdash$ (Γ is recursively enumerable) $\rightarrow$ (Γ is axiomatizable)

Thus, Church's Thesis and Craig's Result together entail that if a set of sentences is effectively enumerable, then it is axiomatizable.

3. C. T., C. R. $\vdash$ (Γ is effectively enumerable) $\rightarrow$ (Γ is axiomatizable)
1,2 *modus ponens*, conditional proof

Finally, an immediate consequence of Gödel's Incompleteness Theorem is that the set of truths of elementary number theory is not axiomatizable. Where "G. I. T." names Gödel's Incompleteness Theorem, and "N" names the set of truths of elementary number theory, we can present this consequence in this manner.

4. G. I. T. $\vdash \neg$(N is axiomatizable).

Thus, Church's Thesis, Craig's Result, and Gödel's Incompleteness Theorem together give us the following.

5. C. T., C. R., G. I. T. $\vdash \neg$(N is effectively enumerable)

[6] All of the results in recursion theory discussed below are proven in (Boolos & Jeffrey, 1989).

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3,4 *modus tollens* (since Γ is schematic in 3)

Assuming Church's Thesis, the set of truths of elementary number theory is not effectively enumerable.

Unfortunately, Dummettian Anti-Realism entails that the set of truths of elementary number theory *is* effectively enumerable. To see why this is the case, note again that Dummett motivates Heyting's identification of mathematical truth with provability by identifying our grasp of the sentences of mathematics with our ability to recognize proofs of those sentences. With this in mind, consider an enumeration of the set of all possible finite sequences of formulas in first order number theory.[7]

$S_1, S_2, S_3 \ldots$

Let "P_n" denote the last formula in the finite sequence "S_n". Now here is a procedure that one who understands claims in elementary number theory can follow to enumerate the set of its truths. For each S_n, check whether S_n is a proof of P_n. If it is not, move on to S_{n+1} and repeat the procedure. If S_n is the first proof of P_n found, then call P_n, "e_1". If S_n is a proof of P_n (but not the first), then call P_n, "e_{i+1}", where e_i is the most recent addition to the list of e's. Then,

$e_1, e_2, e_3 \ldots$

is an enumeration of the truths of arithmetic. Thus, where "M. R." names Dummett's "Manifestation Requirement," that is the identification of our grasp of the meaning of mathematical sentences with the ability to recognize verifications of those sentences,[8] we have

6. M. R. $\vdash$ N is effectively enumerable

But then, the premises in 5. and 6. cannot all be true, so at least one of Church's Thesis, Craig's Result, or Gödel's Theorem is false. Since Craig's Result and Gödel's Theorem are valid (accepted even by intuitionists such as Dummett), it follows that the Dummettian must reject Church's Thesis.

7. C. R., G. I. T., M. R. $\vdash \neg$(C. T.)

5,6 $\neg$ introduction

I conjecture that Dummett had something like this argument in mind.

OBJECTIONS

The only contentious step in the above argument is line 6.

6. M. R. $\vdash$ N is effectively enumerable

One might object that the supposed enumeration ($e_1, e_2, e_3 \ldots$) of mathematical truths constructed in the argument is itself incomplete. Perhaps some truths of number theory are such that there is no proof in the initial enumeration of finite strings ($S_1, S_2, S_3 \ldots$). But then, the "N" in line six would only be an effectively enumerable subtheory of number theory. Since the "N" in line five,

5. C. T., C. R., G. I. T. $\vdash \neg$(N is effectively enumerable)

3,4 *modus tollens* (since Γ is schematic in 3),

[7] Petr Hájek noted (p.c.) that one might interpret (Dummett, 1963) as an argument that the sentences of number theory are not themselves recursively enumerable; and hence for the Dummettian no such enumeration might be possible. As I argue below, in the context of my argument, this would run afoul of Dummett's other commitments. More importantly though, holding that the language of number theory is not recursively enumerable would itself involve denying Church's Thesis, as long as one believed that the human mind possesses a procedure by which to tell whether a given sentence is a sentence of number theory. Thus, this objection blocks the argument only at the price of affirming its conclusion.

[8] Rather than enter into finer points of Dummett exegesis here (see (Cogburn, 1999)), I note that Stewart Shapiro (e.g. (Shapiro 1998)), Crispin Wright (e.g. most of the essays in, (Wright 1987)), and Neil Tennant all interpret M.R. in the manner I have, as sanctioning the above enumeration. For example, Tennant gives the condition as,

> (wpM) For all Φ that the speaker understands: if the condition for the truth of Φ *does* obtain, then the speaker should be able, if given the opportunity to inspect any truth-maker for Φ, to recognize that the condition for Φ's truth obtains, or at least be able to get himself into a position where he can so recognize; but if the condition for the truth of Φ does *not* obtain, then the speaker should be able, *if given the opportunity* to inspect any truth-maker for $\neg\Phi$, to recognize that the condition for Φ's truth does not obtain, or at least be able to get himself into a position where he can so recognize. (Tennant, 1997, 202)

It should also be noted that Chapter 7 of (Tennant, 1997) contains an enumeration not unlike the one I sketch above (for a discussion, see (Cogburn, forthcoming)).

is full first order number theory, the final step would involve an equivocation. While this is perhaps plausible, it is not an objection the Dummettian can make.

Consider a possible true sentence, *P*, in elementary number theory such that none of the S_ns prove *P*. This could be for one of two reasons: (1) *P* is true but absolutely unprovable, or (2) Every proof of *P* involves resources outside of elementary number theory. The first option is clearly unavailable to the Dummettian, who identifies mathematical truth with provability. Since it is an option available to others though, the Dummettian identification should be noted in the premises. Thus, where "V." denotes this verificationist position, the final lines of the argument should read

6.' M. R., V. ├ N is effectively enumerable
7.' C. R., G. I. T., M. R., V. ├ ¬(C. T.)
5,6' ¬ introduction

However, one might still balk at 6' for the second reason given above; perhaps the only proof of *P* involves resources outside of elementary number theory. Thus, while *P* is true and provable, its proof still would not occur in the initial enumeration (S_1, S_2, S_3. . .). While this second option may in fact be correct, it too is anathema for the Dummettian, since when combined with M. R. it commits the Dummettian to an implausible form of holism about grasp of mathematics. For example, suppose that the only possible proof of a number theoretic claim, *P*, involved cutting edge work in topology. Then M. R. would force the Dummettian to say that understanding *P*, a claim in elementary number theory, requires the ability to recognize proofs in cutting edge topology. Besides being extremely implausible in its own right, such holism wreaks great violence to other aspects of Dummett's program. For example, in the classic "What is a Theory of Meaning?" articles and elsewhere,[9] Dummett presents holism about grasp of meaning as the only way to preserve use of traditional truth conditional (versus Heyting Style proof conditional) semantics in the theory of meaning. Thus, Dummett's initial meaning theoretic argument for V and M. R. requires rejecting holism![10] Thus, where "M." denotes the anti-holistic "molecularism" at the heart of Dummett's Anti-Realism, the correct concluding steps to the argument against Church's Thesis should read as the following.

6.'' M. R., V., M.,├ N is effectively enumerable
7.'' C. R., G. I. T., M. R., V., M. ├ ¬(C. T.)
5,6'' ¬ introduction

Since Craig's Result and Gödel's Theorem are both clearly valid, the substantive result is that the Manifestation Requirement, Verificationism, and Molecularism entail that Church's Thesis is false.

A final gambit might involve admitting that the Dummettian anti-holist can't argue that the enumeration of finite strings (S_1, S_2, S_3. . .) is incomplete in the sense that there exists a true mathematical claim not proven by one of the strings. However, one might give other reasons for holding that the enumeration of mathematical truths (e_1, e_2, e_3. . .) is incomplete. Perhaps understanding a discourse only requires being able to recognize a canonical core of proofs for some subset of the set of the truths of that discourse. Then, since we have changed the meaning of "M. R." to a requirement weaker than Dummett's Manifestation Requirement, the enumerated mathematical truths in line 6 might be an axiomatizable subset of the truths of number theory. This, again, would force the argument to equivocate.

Unfortunately, this gambit is also unavailable to the Dummettian. It involves denying the Manifestation Requirement in a way that leaves the Dummettian with no motivation for Verificationism! Remember that Dummett motivates Heyting's identification of truth with provability by identifying our understanding of mathematical claims with our ability to recognize proofs of those claims. If Dummett's position were altered to the one considered here (identifying our understanding of mathematical claims with our ability to recognize proofs of *some* of those claims), there would be no reason to accept Heyting's identification of truth with provability. One would just need to say that all of the members of the subset of mathematical truths that Dummett charges us with recognizing are provable.[11] But

[9]See (Dummett, 1976a), (Dummett, 1976b), and (Dummett, 1991).

[10] For a crucially important discussion of these issues, see (Shapiro, 1998). Shapiro's discussion blocks one possible criticism of my argument. Building on the discussion of (Dummett, 1963) one might argue that the set of finite sequences of sentences of number theory is *not* enumerable. Shapiro shows very clearly how such a response violates Dummettian molecularism.

[11] Tennant (see, for example Chapter 12 of (Tennant, 1997)) interprets Gentzen and Prawitz style normal form proofs as prohibiting this, that is, as showing that a deductive system is kosher in the sense that if a conclusion is provable non-canonically from a set of premises in the system, then that conclusion will also be proven canonically within the system. Göran Sundholm and Thomasz Placek (p.c.) have both stressed how this canonicity requirement plays a similar role as the conservative extension requirement.

this, again, is consistent with the existence of true mathematical claims that have no proofs, *pace* Dummett and Heyting. Therefore, Dummettian Anti-Realism conclusively undermines Church's Thesis.

3.0 CONCLUDING REMARKS

The importance of this result should not be understated. The Dummettian holds that our minds have access to a procedure by which mathematical proofs can be recognized, a procedure by which the truths of mathematics can be enumerated. Yet Gödel's Theorem and Craig's Result entail that the resulting set of sentences is not recursively enumerable. Thus, the Dummettian is committed to a notion of psychological computability that somehow outstrips what a computer can do. The Dummettian must now keep company with Penrose and Lucas, holding that Gödel's Theorem undermines the computational model of mind.[12] This denial of the computational model of mind should be a cause of concern for the Dummettian, as it shows how Dummett's forfeiture of Church's Thesis potentially robs Anti-Realism of the epistemic virtues claimed for it. To see this, remember Paul Benacerraf's classic comments on the respective goodmaking features of Platonist and Constructivist philosophies of mathematics. In "Mathematical Truth" he writes.

It is my contention that two quite distinct kinds of concerns have separately motivated accounts of the nature of mathematical truth: (1) the concern for having a homogeneous semantical theory in which semantics for the propositions of mathematics parallel the semantics for the rest of the language, and (2) the concern that the account of mathematical truth mesh with a reasonable epistemology. It will be my general thesis that almost all accounts of the concept of mathematical truth can be identified with serving one or another of these masters *at the expense of the other*. (Benacerraf, 1983, 403)

Benacerraf's piece is perhaps the most influential article in the philosophy of mathematics because the problem he describes has determined a research agenda for the Platonist and the Constructivist. The Platonist, having a good semantics, needs to address the epistemology of mathematics. Constructivists, having good epistemologies, need to address the semantics, largely by seeing how much traditional mathematics can be made constructively kosher.

In closing I want to suggest that the denial of Church's Thesis is especially damning for the Dummettian because it shows that Dummett's intuitionistic brand of Constructivism is epistemologically problematic. If this is right, then Dummettian Anti-Realism lacks the very property that recommends Constructivism.

To see how this is the case, first note that Gödel's Theorem is *prima facie* problematic for the mere identification of truth with provability (which Dummett gets from Heyting, and which was the premise V. above). For one might think that V. is wholly undermined by the result that any consistent attempted axiomatization of arithmetic will fail to prove some truths of arithmetic. Dummett addresses this issue in "The Philosophical Significance of Gödel's Theorem," where he argues persuasively that the intuitionist's notion of mathematical provability is something that cannot be captured by one axiomatization. So when the follower of Heyting says that mathematical truth is provability, she does *not* intend to contradict Gödel's Theorem and say that there exists a consistent axiomatization of mathematical provability.

As a metaphysical or semantic point about constructive truth this is hardly controversial. However, the epistemic import of Gödel's Theorem in this context has not been appreciated. Remember, the Dummettian does not merely identify mathematical truth with provability, she also (in order to motivate the identification) identifies our grasp of mathematical truth with the ability to recognize such proofs. While this seems to give Dummett a real epistemic advantage over Platonism, one begins to suspect that such an advantage is chimerical.[13] The Platonist cashes out our understanding of mathematics in terms of insight into infinite structures that populate Plato's heaven. In light of Gödel's Theorem, the Dummettian now cashes out our understanding in terms of insight into an infinite set of proofs that have no axiomatic basis.

If this were where things stood, then it would not be clear whether the Platonist's *tu quoque* against the Dummettian had any force. Unfortunately for Dummettians, things do not so stand. As we have shown, the Dummettian can only accommodate Gödel's Theorem at the price of abandoning the computational model of mind. While the Dummettian may be right to criticize Platonism as providing a bad epistemology of mathematics, she herself can now hardly claim to have presented a better one. For the Platonist, the mystery concerns how the mind recognizes Forms that exist neither in space nor time; for the Dummettian, the mystery concerns how the mind recognizes

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Epistemically, the Dummettian only requires that understanders be able to recognize canonical proofs. This is why the Dummettian holds that if a claim is provable, then it must provable canonically.

[12] See (Lucas, 1961), (Penrose, 1989), and (Penrose, 1994). Note that, unlike the Lucas/Penrose argument, my argument does *not* assume that people are consistent or that they can know their own Gödel sentence. In (Cogburn & Megill, in preparation), Jason Megill and I expand on this as well as the argument's relationship to inferentialism more broadly.

[13] For more on the issue of epistemic gain in this context, see (Shapiro, 1993).

members of a set of proofs that has no axiomatic basis. Until Dummettians say more about how their model of mind works detractors will be right to conclude that the Dummettian is replacing one epistemic mystery with another one.

Strangely, Dummett's brand of intuitionism might share more with a certain kind of ethical intuitionism than anybody would have ever imagined. G.E. Moore held that moral terms are not definable, but that this posed no special problem for moral knowledge, because we have a special faculty of moral intuition that can somehow directly discern the moral truth. Given this claim about the undefinability of moral terms, Moorean ethical intuitionists were precluded from saying very much about this special faculty. This, as Alisdair MacIntyre has argued, led to a horrible parody of moral conversation.

How were such questions to be answered? By following Moore's prescriptions in a precise fashion. Do you or do you not discern the presence or absence of the non-natural property of good in greater or lesser degree? And what if two observers disagree? Then, so the answer went, according to Keynes, either the two were focusing on different subject matters, without recognizing this, or one had perceptions superior to the other. But, of course, as Keynes tells us, what was really happening was something quite other: 'In practice, victory was with those who could speak with the greatest apperance of clear, undoubting conviction and could best use the accents of infallibility' and Keynes goes on to describe the effectiveness of Moore's gasps of incredulity and head-shaking, of Strachey's grim silences and of Lowes Dickinson's shrugs. (MacIntyre, 1981, 17)

If MacIntyre is right, it is no accident that those subjected to these gasps, head-shakings, silences, and shrugs concluded that moral judgment was not really judgment at all, but rather the non-rational emoting of one's own feelings. Moorean ethical intuitionism leads to some variety of non-cognitivism or skepticism precisely because the faculty of moral intuition posited is so mysterious as to be not believable.[14] This mystery compels ethical realists to undermine the Moorean reasons that might lead one to be an intuitionist in the first place. Might Dummettian intuitionism now be revealed to be just as unstable, and for the same reasons? [15]

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[14] This point is not just MacIntyre's view, but can be found in nearly any book covering meta-ethics, e.g. (Garner & Rosen, 1967). Note that I am not making a criticism of every species of ethical intuitionism. The idea that we have basic moral intuitions (e.g. "suffering is bad") is *not* in itself problematic. It only becomes problematic when coupled with the claim that moral terms are undefinable, and hence that moral theory is unaxiomatizable. It is this problem besetting Moore's intuitionism that is so similar to the problem Dummett faces.

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