Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-22, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

Formulation of Mathematical Model that Account the Actual Distances Political Party Traversed During Election Campaign

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Abstract

Democracy, which is a form of government in which all eligible citizens have an equal say in the decisions that affect their lives is very crucial to every nation's development. All over the world, political parties are seen as vital institutions for contemporary democratic dispensation and they play a vital role in the democratic process. They are crucial for the organization of modern democracy and are relevant for the expression and manifestation of political pluralism. The strategic role that parties play in any democratic nation means that funding their activities cannot be discounted. Finance is regarded as the most essential resource for political parties (van Biezen, 2003). Yet for too long, commitment to financing of political parties in Ghana has remained rhetoric.

In many ways, political activities involve expenses which should be seen as the necessary and unavoidable cost of democracy. Because money is one of the most essential resources for political parties, which are principal actors of modern democracy, it plays a critical role in the democratic process (van Biezen, 2003). In order for political parties to maintain their party organizations, employ party personnel, conduct election campaigns and communicate with electorates at large, appropriate financial resources need to be available. These leave political parties with no option than to fall on multinational companies or rich individuals to finance their election campaign activities. Allowing multinational companies or rich individuals to finance election campaign is not the best since the financiers would have to be compensated. This could lead to political corruption. It is therefore a necessity and very prudent for political parties to find ways of minimizing cost. The presidential aspirants' taking the optimal route in all of his /her campaign visitation is one surest way of minimizing cost. This is what this thesis seeks to achieve or do. The major component of the cost incurred by the presidential campaign team is attributed to the cost of transportation between constituencies. The need to reduce the cost of campaign requires a critical look at minimizing the cost of transportation.
Keywords: *Mathematical Model, Actual Distances, Political Party Traversed, Election Campaign*

1. INTRODUCTION

The Travelling Salesman Problem (TSP) is an NP-hard problem in combinatorial optimization studied in operations research and theoretical computer science. The Traveling Salesman Problem (TSP) is one of the most widely studied combinatorial optimization problems. Its statement is deceptively simple, and yet it remains one of the most challenging problems in Operational Research. Hundreds of articles have been written on the TSP. The book edited by Lawler et al., (1985) provided an insightful and comprehensive survey of all major research results until date. Given a list of cities and their pairwise distances, the task is to find the shortest possible route that visits each city exactly once. In the Travelling Salesman Problem (TSP), we are given n nodes and for each pair $\{i, j\}$ of distinct nodes, a distance$d_{i,j}$. We desire a closed path that visits each node exactly once (that is a salesman tour) and incurs the least cost, which is the sum of the distances along the path. In the m-TSP problem, m-number of salesmen have to cover the given cities and each city must be visited by exactly one salesman. Every salesman starts from the same city, called depot, and must return at the end of his journey to this city again. The TSP has been a classic problem, which has influenced the emergence of fields such as operations research. Since the 1970s, mounting evidence from complexity theory suggests that the problem is computationally difficult. Exact optimization is NP-hard (Karp, 1990). The Travelling Salesman Problem (TSP) was first formulated as a mathematical problem in 1930 and is one of the most intensively studied problems in optimization. It is used as a benchmark for many optimization methods. Even though the problem is computationally difficult, a large number of heuristics and exact methods are known, so that some instances with tens of thousands of cities can be solved.

The TSP is one of the major success stories for optimization. Decades of research into optimization techniques, combined with the continuing rapid growth in computer speeds and memory capacities, have led to one new record after another. Over the past 33 years, the record for the largest nontrivial TSP instance solved optimality has increased from 318 cities (Crowder and Padberg, 1980) to 2392 cities (Padberg and Rinaldi, 1987) to 7397 cities (Applegate, Bixby, Chvatal, and Cook, 1994). Admittedly, this last result took roughly 3-4 years of CPU- time on a network of machines of the calibre of a SPARCstation 2. (SPARC is a trademark of SPARC international, Inc. and is licensed exclusively to Sun Microsystems, Inc.). However, the branch-and-cut technology developed for these record-setting performances has also had a major impact on the low end of the scale. Problems with 100 or fewer cities are now routinely solved within a few minutes on a workstation (although there are isolated instances in this range that take much longer) and instances in the 1, 000-city range typically take only a few hours (or days).

The Travelling Salesman Problem (TSP) has been studied for the past fifty-two (52) years and many exact and heuristic algorithms have been developed. These algorithms include construction algorithms, iterative improvement algorithms, branch- and -bound (B&B) and branch- and -cut (B&C) exact algorithms and many meta-heuristic algorithms, such as simulated annealing (SA), Tabu Search (TS), Ant Colony (AC) and Genetic Algorithm GA). The method of simulated annealing quickly gives a sub-optional answer, where 'sub-optional' means solution close enough to be the true minimum path. A number of results were developed by Chandra and Tovey (1999) some worst-case and some probabilistic, on the performance of 2- and k-opt local search for the TSP, with respect to both the quality of the solution and the speed with which it is obtained. Indeed, subsequent experiments suggest that when $N > 100$ the elastic net approach does not even outperform 2-opt, which typically gets within 5% of the Held-Karp bound for random Euclidean instances. Perterson (1990) compared the elastic net approach to various others on 200-city instances and obtained results that averaged 6% worse than those found by a version of the M$\ddot{u}$hlenbein et al., (1988) genetic algorithm. Vakhutinsky and Golden (1995) used a hierarchical scheme to speed up the process (and slightly improve tour quality) and were able to handle 500-city random Euclidean instances, but they appear to have found tours that were more than 10% above the Held-Karp bound.

Boeres, de Carvalho, and Barbosa (1992) sped up the elastic net approach in much the same way that Bonomi and Lutton (1984) sped up simulated annealing, using a partition of the unit square into a grid of smaller cells to help narrow their searches. In particular, when computing the force imposed on a vertex by the cities, they restricted attention to just those cities in nearby cells. This enabled them to 1000-city random Euclidean instances, and they claim that there is no significant loss in tour quality. Their tours, however, are 5-10% worse than those found by their implementation of Lin-Kernighan, and they are hence worse than what 2-opt could have provided, given that 2-opt typically only 3% or so behind Lin-Kernighan on random Euclidean instances. Moreover, 2-opt and even Lin-Kernighan would have been substantially faster: for 1000 cities the Boeres et al., (1992) running time is roughly 15, 000 seconds on a SPARCstation, versus an average of less than a second for Lin-Kernighan on our SGI Challenge, a machine that is at most 15 times faster. In their paper, Boeres et al.,(1992) claim significant speed advantages for elastic nets over Lin-Kernighan, but this comparison is based on an implementation of the latter that appears to have a worse than quadratic running time growth rate, as opposed to the decidedly subquadratic running time reported for Lin-Kernighan. An implementation of the Lin-Kernighan heuristic, one of the most effective methods for giving optimal or near optimal solutions for the symmetric TSP is described (Helsgaun,2000) , (Gambardella and Dorigo, 1996). Computational tests show that the implementation is highly valuable. A simulated annealing process starts with the cities connected in a random order, and then considers making random changes in that order. If changing the order of cities leads to a shorter path, we accept that change. If the modification yields a longer path, we give ourselves a certain probability of accepting the modification. The probability is gradually reduced over time in order to rule out shorter and shorter path increase thereby converging toward a path length close to the absolute minimum.

Belisle et al., (1993) discussed convergence properties of simulated annealing algorithms applied to continuous functions and applies these results to hit-and-run algorithms used in global optimization. His convergence properties are consistent with those presented by Hajek(1988) and he provides a good contrast between convergence in probability and the stronger almost sure convergence. Zabinsky et al., (1993) extended this work to an improved hit-and-run algorithm used for global optimization. Fleischer and Jacobson (1996) proposes cybernetic optimization by simulated annealing as a method of parallel processing that accelerated the convergence of simulated annealing to the global optima. Fleischer (1999) extended the theory of cybernetic optimization by simulated annealing into the continuous domain by applying probabilistic feedback control to the generation of candidate solutions. The probabilistic feedback control method of generating candidate solutions effectively accelerates convergence to a global optimum using parallel simulated annealing on continuous variable problems. Locatelli (1996) presented convergence properties for a class of simulated annealing algorithms for continuous global optimization by removing the restriction that the next candidate point must be generated according to a probability distribution whose support is the whole feasible set.

Siarry et al., (1997) studied simulated annealing algorithms for globally minimizing functions of many-continuous variables. Their work focuses on how high- dimensionality can be addressed using variables discretization, as well as considering the design and implementation of several complementary stopping criteria. Yang (2000) and Locatelli (2000) also provided convergence results and criteria for simulated annealing applied to continuous global optimization problems. Kiatsupaibul and Smith (2000) introduced a general purpose simulated annealing algorithm to solve mixed integer linear programmes. Aarts and Korst (1989), in Boltzmann machine implementation, used extra neutrons to insure that locally optimal solutions correspond to connected graphs. Joppe, Cardon and Bioche (1990) propose something similar, with a second level of neutrons that can trigger changes to the external inputs for the first level. Neither approach seems to have been tried on instances with more than 30 cities, however. A more effective (if less neutral) approach seems to be to invoke the classical patching algorithms of Karp (1977). This was proposed and implemented by Xu and Tsai (1991), handled up to 150-city instances. Before switching to Karp's patching heuristic, Xu and Tsai perform five successive neutral net runs, each adding penalty terms to the energy function so as to inhibit the creation of the subtours seen in the previous rounds. Some of the well-known tour construction procedures are the nearest neighbour procedure by Rosenkratz et al., (1977), the Clark and Wright savings' algorithm, the insertion procedures, the partitioning approach by Karp and the minimal spanning tree approach by Christofides.

The branch exchange is perhaps the best known iterative improvement algorithm for the TSP. The 2-opt and 3-opt heuristics were described in Lin. Lin and Kernighan (1973) made a great improvement in quality of tours that can be obtained by heuristic methods. Even today, their algorithm remains the key ingredient in the successful approaches for finding high quality tours and is widely used to generate initial solutions for other algorithms or developed a simplified edge exchange procedure requiring only $O(n^2)$ operations at each step, but producing tour nearly as good as the average performance of 3-opt algorithm. Determining a minimum-cost arborescence rooted at vertex r, and finding the minimum-cost arc entering vertex r. The first problem is easily solved in $O(n^2)$ time (Tarjan, 1977). This relaxation can be used in conjunction with Lagrangian relaxation. However, on asymmetric problems, the Assignment Problem (AP) relaxation would appear empirically superior to the r- arborescence relaxation (Balas and Toth, 1977).

Christofides (1970) and Held and Karp (1971) were among the first to propose a TSP algorithm based on this relaxation. Improvements and refinements were later suggested by Helbig Hansen and Krarup (1974), Smith and Thompson (1977), Volgenant and Jonger (1982), Gavish and Srikanth (1986), and Carpaneto, Fischetti and Toth (1989). One of the earliest exact algorithms is by Dantzig et al., (1954), in which Linear Programming (LP) relaxation is used to solve the integer formulation by suitably choosing linear inequality to the list of constraints continuously. Branch and bound (B & B) algorithm are widely used to solve the TSP's. Several authors have proposed B & B algorithm based on assignment problem (AP) relaxation of the original TSP formulation. These authors include Eastrnan (1958), Held and Karp (1970), Smith et al.,(1977), Carpaneto and Toth (1980), Balas and Christofides (1981). Some Branch and Cut (B & C) based exact algorithms were developed by Crowder and Padberg (1980), Padberg and Hong (1980), Grötschel and Holland (1991). The lower- bounding procedure described by Fischetti and Toth was embedded within the Carpaneto and Toth (1980) branch-and-boundalgorthm on a variety of randomly generated problems and on some problems described in the literature. The success of the algorithm depends on the type of problem considered. For the easiest problem type, the authors report having solved 2000- vertex problems in an average time of 8329 seconds on an Hp 9000/840 computer.

Apart from the exact and heuristic algorithms stated above, meta-heuristic algorithms have been applied successfully to the TSP by a number of researchers. Simulated Annealing (SA) algorithms for the TSP were developed by Bonomi and Lutton, Golden and Skiscim(1986) and Nahar et al., (1989) etc. Tabu search meta-heuristic algorithms for TSP have been proposed by Knox and Fiechter (1994). The Ant Colony (AC) is a relative new meta-heuristic algorithm which is applied successfully to solve the TSP, some work based on SA technology was reported by Dorigo et al., (1996). Genetic algorithms for the TSP were reported by Goefenstetle et al., (1985). Applegate et. al., (1994) solved a travelling salesman problem which models the production of printed circuit boards having 7, 397 holes (cities), and in 1998, the same authors solved a problem over the 13, 509 largest cities in the U.S. For problems with large number of nodes as cities the TSP becomes more difficult to solve. In Homer's Ulysses (1993) problem of a 16 city travelling salesman problem, one finds that there are 653, 837, 184, 000 distinct routes (Grötschel and Padberg, 1993). Enumerating all such roundtrips to find the shortest one took 92 hours on a powerful workstation. The TSP and its solution procedures have continued to provide useful test grounds for many combinatorial optimization approaches. Classical local optimization techniques Rossman (1958); Applegate et al., (1999); Riera-Ledesma, (2005); Walshaw, (2002); Walshaw (2001) as well as many of the more recent variants on local optimization, such as simulated annealing by Tian and Yang (1993), tabu search by Kolohan and Liang, (2003), neural networks by Potvin, (1996) and genetic algorithms have all been applied to this problem, which for decades has continued to attract the interests of researchers.

Although a problem statement posed by Karl Menger on February 5, 1930, at a mathematical colloquium in Vienna, is regarded as a precursor of the TSP, it was Hassle Whitney, in 1934, who posed the travelling salesman problem in a seminar at Princeton University (Flood, 1956).

In 1949, Robinson, with an algorithm for solving a variant of the assignment problem is one of the earliest references that use the term "travelling salesman problem" in the context of mathematical optimization. (Robinson, 1949) , however, a breakthrough in solution methods for the TSP came in 1954, when Dantzig et al., (1954) applied the simplex method (designed by George Dantzig in 1947) to an instance with 49 cities by solving the TSP with linear programming. There were several recorded contributions to the TSP in 1955. Heller, (1955) discussed linear systems for the TSP polytope, and some neighbour relations for the asymmetric TSP polytope. Also Kuhn, (1955) announced a complete description of the 5-city asymmetric TSP polytope. Morton and Land (1955) presented a linear programming approach to the TSP, alongside the capacitated vehicle routing problem. Furthermore, Robacker (1955) reported manual computational tests of some 9 cities instance using the Dantzig-Fulkerson-Johnson method, with average computational times of about 3 hours. This time became the benchmark for the next few years of computational work on the TSP (Robacker, 1955).

Flood (1956) discussed some heuristic methods for obtaining good tours, including the nearest-neighbour algorithm and 2-opt while Kruskal, (1956) drew attention to the similarity between the TSP and the minimum-length spanning tress problem. The year 1957 was a quiet one with a contribution from Barachet, (1957) described an enumeration scheme for computing near-optimal tours. Croes (1958) proposed a variant of 3-opt together with an enumeration scheme for computing an optimal tour. He solved the Dantzig-Fulkerson-Johnson 49-city example in 70 hours by hand. He also solved several of the Robacker examples in an average time of 25 minutes per example. Bock (1958) describes a 3-opt algorithm together with an enumeration scheme for computing an optimal tour. The author tested his algorithm on some 10-city instance using an IBM 650 computer. By 1958, work related to the TSP had become serious research to attract Ph.D. students. A notable work was a Ph.D. thesis Eastman, (1958) where a branch-and-bound algorithm using the assignment problem to obtain lower bounds was described. The algorithm was tested on examples having up to 10 cities. Also that same year, Rossman and Twery (1958) solved a 13-city instance using an implicit enumeration while a step-by-step application of the Dantzig-Fulkerson-Johnson algorithm was also given for Barachet's 10-city example. Bellman (1960) showed the TSP as a combinatorial problem that can be solved by dynamic programming method.

Miller et al., (1960) presented an integer programming formulation of the TSP and its computational results of solving several small problems using Gomory's cutting-plane algorithm was reported. Lambert (1960) solved a 5-city example of the TSP using Gomory cutting planes. Dacey (1960) reported a heuristic, whose solutions were on average 4.8 per cent longer than the optimal solutions. TSP in 1960 achieved national prominence in the United States of America when Procter and Gamble used it as the basis of a promotional context. Prizes up to $10,000.00 were offered for identifying the most correct links in a particular 33-city problem. A TSP researcher, Gerald Thompson of Carnegie Mellon University won the prize in Applegate et al., (2007). Muller-Merbach (1961) proposed an algorithm for the asymmetric TSP, the dynamic programming approach gained attention. Gonzales solved instances with up to 10 cities using dynamic programming on an IBM 1620 computer by Gonzales, (1962). Similarly, Held and Karp (1962) described a dynamic programming algorithm for solving small instances and for finding approximate solutions to larger instances. Little et al,. (1963) coined the term branch-and-bound. Their algorithm was implemented on an IBM 7090 computer and they gave some interesting computational tests including the solution of a 25-city problem that was in the Held and Karp test set. Their most cited success is the solution of a set of 30-city asymmetric TSP'S having random edge lengths. In an important paper (Lin, 1965): a heuristic method for the TSP was published.
The author defined k-optional tours, and gave an efficient way to implement 3-opt, extending the work of Croes (1958) with computational results given for instances with up to 105 cities.

The year 1966 was another fruitful one for the TSP in terms of published works. Roberts and Flores (1966) described an enumerative heuristic and obtained a tour for Karg and Thompson's 57-city example, having cost equal to the best tour found by Karg and Thompson. Also, in a D.Sc. thesis at Washington University, St. Louis, Shapiro (1966) describes an algorithm similar to Eastman's branch-and-bound algorithm. Gomory (1966) gave a very nice description of the methods contained in Dantzig et al. (1954), Held and Karp (1962) and Little et al. (1963). Similarly, in Lawler and Wood (1966) descriptions of the branch-and-bound algorithms of Eastman 1958 and Little et al. (1963) were given. The authors suggested the use of minimum spanning tress as a lower bound in a branch-and-bound algorithm for the TSP. Bellmore and Nemhauser (1968) presented an extensive survey of algorithms for the TSP. They suggested dynamic programming for TSP problems with 13 cities or less, Shapiro's branch-and-bound algorithm for larger problems up to about 70-100 and Shen Lin's '3-opt' algorithm for problems that cannot be handled by Shapiro's algorithm. Raymond (1969) is an extension to Karg and Thompson's 1964 heuristic for the TSP where computational results were reported for instances having up to 57 cities. Held and Karp (1970) introduced the 1-tree relaxation of the TSP and the idea of using node weights to improve the bound given by the optimal 1-tree. Their computational results were easily the best reported up to that time. Another notable work on the TSP in the 70s is the S. Hong, Ph.D. Thesis, at The Johns Hopkins University in 1972 written under the supervision of Bellmore, and the work was the most significant computational contribution to the linear programming approach to the TSP since the original paper of Dantzig et al., (1959).

The Hong's (1972) algorithm had most of the ingredients of the current generation of linear-programming based algorithms for the TSP. He used a dual LP algorithm for solving the linear-programming relaxations; he also used the Ford-Fulkerson max-flow algorithm to find violated sub-tour inequalities. The algorithm of Held and Karp (1971) was the basis of some major publications in 1974. In one case, Hansen and krarup (1974) tested their version of Held-Karp (1971) on the 57-city instance of Karg and Thompson 1964 and a set of instances having random edge lengths. In 1976 a linear programming package written by Land and Powell was used to implement a branch-and-cut algorithm using sub-tour inequalities. Computational results for the 48-city instance of Held and Karp and the 57-city instance of Karg and Thompson (1964) were given. Smith and Thompson (1977) presented some improvements to the Held-Karp algorithm tested their methods on examples which included the 57-city instance of Karg and Thompson 1964 and a set of ten 60-city random Euclidean instances. In 1979, Land described a cutting-plane algorithm for the TSP. The decade ended with a survey on algorithms for the TSP and the asymmetric TSP in Buckard, (1979).

Crowder and Padberg (1980) gave the solution of a 318-city instance described in Lin and Kernighan (1973). The 318-city instance remained until 1987 as the largest TSP solved. Also, in 1980, Grötschel gave the solution of a 120-city instance by means of a cutting-plane algorithm, where sub-tour inequalities were detected and added by hand to the linear programming relaxation in Grötschel (1980). In 1982, Volgenant and Jonker described a variation of the Held-Karp algorithm, together with computational results for a number of small instances by Volgenant and Jonker (1982). A very important work of 1985 is a book (Lawler et al., 1985) containing several articles on different aspects of the TSP as an optimization problem. Padberg and Rinaldi (1987) solved a 532-city problem using the so-called branch and cut method. The approach for handling the sub-tours elimination constraints of the TSP integer LP is another area for re-examination. Researchers have identified the issue of feasibility or sub-tour elimination as very crucial in the formulation of the TSP or similar permutation sequence problem. "No one has any difficulty understanding sub-tours, but constraints to prevent them are less obvious "Radin, (1998).

Methodologies or theoretical basis for handling these constraints within the context of algorithm development has been the basis of many popular works on the TSP. A classic example of this approach is in Crowder and Padberg (1980) where a linear programming relaxation was adopted such that if the integral solution found by this search is not a tour, then the sub-tour inequalities violated by the solution are added to the relaxation and resolved.

Grötschel (1980) used a cutting-plane algorithm, where cuts involving sub-tour inequalities were detected and added by hand to the linear programming relaxation. Hong (1972) used a dual LP algorithm for solving the linear-programming relaxations, the Ford –Fulkerson max-flow algorithm, for finding violated sub-tour inequalities and a branch-and-bound scheme, which includes the addition of sub-tour inequalities at the nodes of the branch-and-bound tree. Such algorithms are now known as "branch-and-cut". The problem of dealing with sub-tour occurrences algorithm development has been a major one in the TSP studies in the literature. The works in the 1990's were mostly application in nature. A large number of scientific/engineering problems and applications such as vehicle routing, parts manufacturing and assembly, electronic board manufacturing, space exploration, oil exploration, and production job scheduling, etc. have been modeled as the Machine Setup Problem (MSP) or some variant of the TSP are found in (AL-Haboub-Mohamad and Selim Shokrik (1993), Clarker and Ryan (1989), Keuthen (2003), Kolohan and Liang (2000), Mitrovic-Minic and Krishnamurti, (2006)).

One of the ultimate goals in computer science is to find computationally feasible exact solutions to all the known NP-Hard problems; a goal that may never be reached. Feasible exact solutions for the TSP have been found, but there are restrictions on the input sizes. An exact solution was found for a 318-City problem by Crowder and Padberg in (1980). The basic idea in achieving this solution involves three phases. In the first phase, a true lower bound on the optimal tour is found. In the second phase, the result in the first phase is used to eliminate about ninety-seven per cent of all the possible tours. Thus, only about three per cent of the possible tours need to be considered. In the third phase, the reduced problem is solved by brute force. This solution has been implemented and used in practice. Experimental results by Apple Gate et al.,(1998) showed that running this algorithm, implemented in the C programming language and executed on a 400MHz machine, would produce a result in 24.6 seconds of running time. Other exact solutions have been found. As mention in 1998, a 120-city problem by Grotschel (1980), a 532-city problem by Padberg and Rinaldi (1987). However, none of the algorithms that provide an exact solution for input instances of over a thousand cities are practical for everyday use. Even with today's super computers, the execution time of such exact solution algorithms for TSPs involving thousands of cities could take days.

Computer hardware researchers have been making astonishing progress in manufacturing evermore powerful computer chips. Moores Law in (http://en. Wikipedia.org/wiki/Moore's_law), which states that the number of transistors that can fit on a chip will double after every 18 months, has held ground since 1965. This basically means that computing power has doubled every 18 months since then. Thus, we have been able to solve larger instances of NP-hard problems, but algorithm complexity has still remained exponential. Moreover, it is highly speculated that this trend will come to an end because there is a limit to the miniaturization of transistors. Presently, the sizes of transistors are approaching the size of atoms. With the speeds of computer processors rounding the 5GHz mark, and talks about an exponential increase in speeds of up to 100GHz (http://en. Wikipedia.org/wiki/Moore's law), one might consider the possibility of us exceeding any further need of computational performance. However, this is not the case. Although computing speeds may increase exponentially, they are, and will continue to be, surpassed by the exponential increase in algorithmic complexity as problem sizes continue to grow. Moore's law may continue to hold true for another decade or so, but different methods of computing are being researched.

2. METHODOLOGY

Combinatorial optimization is the process of finding the globally optimal configuration of discrete variables with respect to some function of the variables. Many combinatorial optimization problems are very difficult and are NP- hard. A large number of combinatorial problems are of practical interest and importance, examples are the Traveling Salesman Problem, timetabling, routing and scheduling, and layout and placement problems. In the Travelling Salesman Problem (TSP), we are given *n* nodes and for each pair $\{i, j\}$ of distinct nodes, a distance$d_{i,j}$. We desire a closed path that visits each node exactly once (that is a salesman tour) and incurs the least cost, which is the sum of the distances along the path. This task of visiting the constituencies can be modeled as a classical Traveling Salesman problem. An aspirant wishes to visit the twenty three (23) constituencies in the central region of Ghana. Minimizing the total travel distance to each constituency for campaign purposes saves time and reduces the cost of the campaign trip. The TSP is to find the shortest circuitous path connecting n-number of cities. This means that a salesman following that path would visit each city only once. For smaller number of cities (n≥ 4) there is the possibility of considering even a manual solution of the TSP. However when the number of cities is large (n≥ 10) the method of manual computation cannot be applied and computerized methods of finding all routed length can be impractically slow. Manual computation is not practical because the number of circuits grows so fast that even for n=25 cities, it would take longer than the age of the universe (~ 10 billion years) to check all paths at a rate of one million paths per second since there are $(\frac{1}{2})n!$ paths according to Gato(1991).

A presidential aspirant's visit to a constituency comes with a lot of benefits to both the aspirant and the electorate. Some of these benefits are catalogued below.

- It affords the voters an opportunity to have knowledge of issues. Of all the information voters obtain through the mass media during a presidential campaign, knowledge about where the candidates stand is most vital by Patterson and McClure (1976).
- It gives the aspirant a fair idea of the specific challenges in the various constituencies and assures the electorate as to how such challenges would be addressed.
- The aspirant seizes the opportunity to deliver his/her campaign message or policies and discuss his/her policy positions.
- Some of the electorates get to see the aspirant for the first time as he/she is introduced to the electorates.
- The visit enables the aspirant to canvass for votes.
- Party foot soldiers are motivated to hit the campaign trail even in the absence of the aspirant.

The TSP has provided a test bed for the development of algorithm such as the nearest neighbour rule that approximate optimal solutions of combinatorial optimization problems and on the other hand has prompted questions concerning the performance of such algorithms. The versatility of the application of TSP is briefly discussed below. The Vehicle Routing Problem (VRP) is the m-TSP, where a demand is associated with each city or customer and each vehicle has a certain capacity. As a further constraint to the minimization of the distance covered in a typical TSP, the VRP also considers the minimization of the number of vehicles used. The constraints may include the available fuel capacity of each vehicle and available time windows for customers. TSP based algorithms have been applied in this kind of problem and may also be applied to routing problems in computer networks. (Gerard 1994).

Intrinsically, the VRP is a spatial problem. During the last few decades, however, temporal aspects of routing problems have become increasingly important. Specific examples of problems with time windows include bank deliveries, postal deliveries, industrial refuse collection, school-bus routing and situations where the customer must provide access, verification, or payment upon delivery of the product or service. In these problems customers could be served only during certain hours or the day, such as office hours or the hours before the opening of a shop. For example, a warehouse may only accept deliveries between 10:00 am and 4:00pm.Much attention however has been given to the Vehicle Routing Problem with Time Windows (VRPTW). The time windows can be hard or soft. In the hard time window case, if a vehicle arrives too early at a customer, it is permitted to wait until the customer is ready to begin service. However, a vehicle is not permitted to arrive at a customer after the latest time to begin service. The field of multi-objective optimization is attracting more and more attention, notably because it offers new opportunities

Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-22, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

for defining problems, Jozefowiez (2008). In contrast, in the soft time window case, the time windows can be violated at a cost. The VRP can be modeled using the binary variables x_{nm}^{v} and y_{n}^{v} according to Goel (2006). x_{nm}^{v} indicates whether $m \in \mathbb{N}$ is visited immediately after node $n \in \mathbb{N}$ by vehicle $v \in V$ $(x_{nm}^{v} = 1)$ $or\ not$ $(x_{nm}^{v} = 0)$. y_{n}^{v} indicates whether node $n \in \mathbb{N}$ is visited by vehicle $v \in V$ (y_{n}^{v} =1) or not (y_{n}^{v} = 0). For each $n \in \mathbb{N}$ the VRP contains the variables t_n and P_n. If node $n \in \mathbb{N}$ is visited by a vehicle, t_n specifies the arrival time and P_n specifies the current load of the vehicle. If no vehicle visits node $n \in \mathbb{N}$, both t_n and P_n are without meaning. The contribution of each vehicle $v \in V$ to the objective function is

$$\sum_{o \in O} y_{n(0,1)}^{v} P_o - \sum_{(n,m) \in A} x_{nm}^{v} C_{nm}^{v}$$

The first term represents the accumulated revenue of served orders; the second term represents the accumulated costs for the vehicle movements.

The VRP is maximize

$$\sum_{v \in V} \Big(\sum_{o \in O} y_{n(0,1)}^{v} P_o - \sum_{(n,m) \in A} x_{nm}^{v} C_{nm}^{v}\Big) \quad subject\ to$$

$$\sum_{(n,m) \in A} x_{nm}^{v} = \sum_{(m,n) \in A} x_{mn}^{v}\ for\ all\ v \in V, n \in \mathbb{N}$$

$$y_{n}^{v} = \sum_{(n,m) \in A} x_{nm}^{v}\ \ for\ all\ v \in V, n \in \mathbb{N}$$

$$\sum_{v \in V} y_{n}^{v} \leq 1\ for\ all\ n \in \mathbb{N}.$$

3. ANALYSIS

To satisfy the constraints (2) and (3) we choose the random

Initial tour $(x^0) = 22 - 2 - 5 - 6 - 17 - 8 - 7 - 22 - 10 - 9 - 11 - 20 - 12 - 21 - 13 - 15 - 14 - 23 - 18 - 4 - 3 - 1 - 19 - 22$

From objective function (1) the initial distance= $d(x^0) = \mathrm{d}(22,2) + \mathrm{d}(2,5) + \mathrm{d}(5,6) + \mathrm{d}(6,17) + \mathrm{d}(17,8) + \mathrm{d}(8,7) + \mathrm{d}(7,22) + \mathrm{d}(22,10) + \mathrm{d}(10,9) + \mathrm{d}(9,11) + \mathrm{d}(11,20) + \mathrm{d}(20,12) + \mathrm{d}(12,21) + \mathrm{d}(21,13) + \mathrm{d}(13,15) + \mathrm{d}(15,14) + \mathrm{d}(14,23) + \mathrm{d}(23,18) + \mathrm{d}(18,4) + \mathrm{d}(4,3) + \mathrm{d}(3,1) + \mathrm{d}(1,19) + \mathrm{d}(19,22) = 2031.2\mathrm{km}$

The initial temperature is taken to be $(T_o) = 4069.00$, $\alpha = 0.99$

Temperature is updated by using the formula $T_{k+1} = \alpha T_k$ where k is the number of iteration.

Stop when $T \leq 42.03$

Simulated annealing algorithm was used to obtain the final solution. Probook hp laptop computer (CORE i3) was used in finding the solution after 1339 iterations in 102.367856 seconds. The execution time varied with the number of iterations.

3.1 Results

After performing 1339 iterations, the optimal tour= $2 - 6 - 9 - 14 - 21 - 16 - 17 - 15 - 18 - 13 - 11 - 7 - 3 - 12 - 19 - 20 - 10 - 8 - 22 - 5 - 4 - 23 - 1 - 2$

Thus,

$$\begin{aligned} &= \mathrm{d}(2,6) + \mathrm{d}(6,9) + \mathrm{d}(9,14) + \mathrm{d}(14,21) + \mathrm{d}(21,16) + \mathrm{d}(16,17) + \mathrm{d}(17,15) + \mathrm{d}(15,18) + \mathrm{d}(18,13) \\ &\quad + \mathrm{d}(13,11) + \mathrm{d}(11,7) + \mathrm{d}(7,3) + \mathrm{d}(3,12) + \mathrm{d}(12,19) + \mathrm{d}(19,20) + \mathrm{d}(20,10) + \mathrm{d}(10,8) \\ &\quad + \mathrm{d}(8,22) + \mathrm{d}(22,5) + \mathrm{d}(5,4) + \mathrm{d}(4,23) + \mathrm{d}(23,1) + \mathrm{d}(1,2) = 786\mathrm{k} \end{aligned}$$

There was no change in the last ten iterations for the optimal. The tour optimal tour is therefore as follows:
Elmina→ Essarkyir→ Apam→ Winneba→ Potsin→ Awutu Breku→ Kasoa→ Agona Swedru→ Agona Nsaba→ Afransi→ Asikuma→ Ajumako→ Saltpond→ Assin Foso→ Dunkwa-on-Offin→ Diaso→ Twifo Praso→ Jukwa→ Assin Breku→ Nsuaem Kyekyewere→ Abura Dunkwa→ Abura(Cape Coast) → Old Hospital Hill(Cape Coast) →Elmina

4. CONCLUSION

Simulated annealing is a heuristic- based search algorithm, motivated by an analogy of physical annealing in solids. It is capable of solving combinatorial optimization problem. The method of simulated annealing has been used to find the global minima cost configuration for NP- complete problems with many local minima. The Simulated annealing algorithm can be a useful tool which is applied to hard combinatorial problems like the TSP. Using simulated annealing as a method in solving the symmetric TSP model has proved that it is possible to converge to the best solution. We conclude that the objective of finding the minimum tour from the symmetric TSP model by the use of simulated annealing algorithm was successfully achieved. The study shows clearly that, any presidential aspirant who visits the Central Region must visit the constituencies in the order below to minimize cost. The order is as follows: Elmina→ Essarkyir→ Apam→ Winneba→ Potsin→ Awutu Breku→ Kasoa→ Agona Swedru→ Agona Nsaba→ Afransi→ Asikuma→ Ajumako→ Saltpond→ Assin Foso→ Dunkwa-on-Offin→ Diaso→ Twifo Praso→ Jukwa→ Assin Breku→ Nsuaem Kyekyewere→ Abura Dunkwa→ Abura(Cape Coast) → Old Hospital Hill(Cape Coast) →Elmina.

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