Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-35, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

Assignment of Vehicles to Routes by Latex Foam Rubber Products Limited

[1]Samuel Amoako, [1]Frank Osei Frimpong, [3]Peter Kwasi Sarpong
[1]Akrokerri College of Education, [1]Kumasi Technical University,
[3]Kwame Nkrumah University of Science and Technology
Email: samkamoako2016@gmail.com | oseifrankfrimpong@yahoo.co.uk | Kp.sarp@yahoo.co.uk

Abstract

The problem of distributing goods from depots to consumers plays an important role in the management of many distribution systems, and therefore when it is programmed efficiently it may yield significant savings. In a typical distribution system, trucks provide delivery and pick-up services to customers that are scattered geographically in a given area. In many of its applications, the common objective of distribution is to find a set of routes for such trucks, satisfying a variety of constraints, so as to minimize the total distribution cost. Most of the manufacturing companies in Ghana utilize trucks to transport their products to their customers. The general problem in such a situation is how to assign a particular truck to a route to minimize the total transportation cost whilst satisfying route and the available constraints to serve the company's customers with the demand for some commodity. The Truck Assignment Problem, which is one of the logistics network problems, concerns the determination of the type of truck to be assigned to a particular route to minimize the total number of gallons of fuel required per trip.

In this work, we use a solution procedure based on Munkres Assignment Algorithm for optimal assignment of non-homogenous fleet of trucks to a given set of routes, where Latex Foam Rubber Products Limited-Kumasi, distributes its products to its customers. The problem of assigning resources such as vehicles to task over time arises in a number of applications in transportation. In the field of freight transportation, truckload motor carriers, railways and shipping companies have to manage fleets of containers (trucks, boxcars) that move one load at a time, with orders arriving continuously over time. In the passenger arena, taxi companies and companies that manage fleets of business jets have to assign vehicles (taxicabs or jets) to move customers from one location to the next (Michael Z. Spivey and Warren Powell, 2003). Ahuja, Magnanti and Orhin (Hartvigsen et al., 1999) provide an excellent review of applications of the assignment problem. Among the applications, they listed are personnel assignments, scheduling on parallel machines, pairing stereo speakers and vehicle and crew scheduling. Other applications include posting military servicemen, airline commuting and classroom assignment.

Keywords: *Vehicles Routes, Assignment of Vehicles, Munkres Assignment Algorithm*

1. INTRODUCTION

1.1 Production and Distribution in earlier time

The term *Production* is defined by Economists as the total physical and mental efforts which satisfy human wants. That is, production covers virtually all activities which directly or indirectly satisfy human wants. However, the term as used in production management refers to the transformation of raw materials into finished or semi-finished products (Mahmoud, 1996). *Distribution,* on the other hand, concerns the series of activities and institutions which ensure the transfer of goods from the producer to the market. It basically involves the transfer or movement of goods from the producer to the consumer at the right time and at the appropriate place. Before the Industrial Revolution (Clarke et al., 1998), most goods were produced either by household or by guilds. There were many households involved in the production of marketable goods. Most of the goods that were produced by these households were things that involved cloth, textiles, clothing as well as art (Wiesner-Hanks, 2006) and tapestries (Jardine, 1996). These would be produced by the households, or by their respective guilds. It was even possible for guilds and merchants to outsource into more rural areas, to get some of the work done. The merchants would bring the raw materials to the workers, who would then make the goods. For example, young girls would be hired to make silk, because they were the only people believed to have hands dexterous enough to make the silk properly. Other occupations such as knitting, a job that was never organized into guilds, could easily be done within the household (Wiesner-Hanks, 2006). Guild work could be contracted to the households for women and children, as well as the men would be involved with production of goods. The income of the household became dependent upon the quality and the quantity of everyone's work (De Vries, 1994). Even if people were not working for an individual guild they could still supply and make items not controlled by the guilds. These would be small, but necessary items like wooden dishes, or soaps (Wiesner-Hanks, 2006). So, basically, much of production was done by, or for, guilds. This would indicate that much of what was done was not done for one individual household, but for a larger group or organization. Before the Industrial Revolution the household was the major site of production, and could be comparable to a factory. However, things were to change a bit during the Industrial Revolution. There was a shift in the running of the household. The everyday goods and products used by the household would slowly shift from mostly home-made to mostly "commercially produced goods". At the same time, the women would obtain jobs outside the household (De Vries, 1994). This is also seen within the context of the *Industrial Revolution* where women would often find small jobs to help supplement their husband's wages (Ross, 1993). This would demonstrate the gradual movement away from the household as a centre of production.

1.2 Industrial Revolution

The Industrial Revolution was a period in the late 18th and early 19th centuries when major changes in agriculture, manufacturing, production, and transportation had a profound effect on the socioeconomic and cultural conditions in Britain. The changes subsequently spread throughout Europe, North America, and eventually the world. In the later part of the 1700s there occurred a transition in parts of Great Britain's previously manual- labor-based economy towards machine-based manufacturing. It started with the mechanization of the textile industries, the development of iron-making techniques and the increased use of refined coal. Trade expansion was enabled by the introduction of canals, improved roads and railways. The introduction of steam power fueled primarily by coal, wider utilization of waterwheels and powered machinery (mainly in textile manufacturing) underpinned the dramatic increases in production capacity. The development of all-metal machine tools in the first two decades of the 19th century facilitated the manufacture of more production machines for manufacturing in other industries. The effects spread throughout Western Europe and North America during the 19th century, eventually affecting most of the world. The impact of this change on society was enormous (Wikipedia, 2009). The Industrial Revolution marked a major turning point in human society; almost every aspect of human life was eventually influenced in some way.

1.3 Production and Distribution Today

The revolutions in transportation and communications technologies have increased the extent of the U.S. domestic markets over the last two centuries. Moreover, the expansion of markets is associated with major changes in the course of American economic history. The introduction of canals in the late eighteenth and the early nineteenth centuries is credited with increasing the levels of inventive activity and triggering industrialization (Sokoloff, 1988). Households became less self-sufficient and became specialized consumer-labourers; firms that specialized in the production of various goods emerged in great numbers. The division of labour within firms led to a re-organization of production and increased levels of productivity (Sokoloff, 1984a, 1984b). In the late 18th and the early 19th centuries, the expansion of the U.S. domestic markets and industrialization caused a rapid decline in household production and a proliferation of specialized manufacturing firms in the American economy (Kim, 2000). In this period, the industrial structure was composed of single-unit firms who specialized in the production of manufacturing goods and wholesale merchants and retail store owners who distributed these goods. Since the manufacturing firms typically specialized in a narrow line of products, it was simply too costly for them to market their products directly to consumers. In this setting, the wholesale merchants, who bought and sold sufficient lines of products, were able to lower the costs of transactions more efficiently. The wholesale merchants were not only able to collect information on various manufacturers by locating in major cities but were also able to collect information on rural consumer demand through the use of sales agents who traveled to rural country stores. In this period, most consumers were able to judge the quality of most products upon visual inspection. However, according to Kim, for some goods, they relied on the local producers' and retail merchants' reputation for honesty.

In the late nineteenth century, with advances in science and technology, it became increasingly difficult for consumers to discern the quality of products which they consumed. As incomes rose, consumers purchased a growing number of products for which they lacked basic knowledge to discern quality. Moreover, Kim indicated that, even the manufacturing processes of the most basic of products such as food became so sophisticated that consumers no longer had enough knowledge to discern whether a product was healthy or poisonous. Finally, as regional domestic markets became increasingly integrated between the late 19th and the early 20th centuries geographic specialization in economic activities increased (Kim, 1995).

1.4 Channels of Distribution in Ghana.

Distribution could be broadly classified into Direct and Indirect distributions. There is direct distribution if the producer supplies the product directly to the consumer without the use of an intermediary or middle man. Indirect distribution involves the use of intermediaries or middlemen and retailers to make the product available to the consumer. According to Mahmoud (1996) there are three main channels of distribution of goods in Ghana. These are from the

i. *Producer* to *Consumer*, where the producer sells directly to the consumer,
ii. *Producer* to the *Retailer* and from the *Retailer*tothe *Consumer*, where the wholesaler is by passed and the producer deals directly with the retailer, and
iii. *Producer* to the *Wholesaler*, from the *Wholesaler* to the *Retailer* and from the *Retailer* to the *Consumer*, where the wholesaler buys in bulk from the producer and stores the goods for later resale to retailers.

Channels of Distribution Used for Industrial Products in Ghana: Industrial producers or sellers in Ghana today use four main channels to distribute their products in the country (Mahmoud, 1996). These are from the:

- ***Producer to Consumer*****:** Most industrial producers such as Tema Steel Works, the Timber Processing organizations and Vehicle or Machine component manufacturing companies use this channel of distribution

- ***Producer to Industrial distributor (customer)*****:** Some producers of industrial products use industrial distributors to market their products in Ghana.

- ***Producer to an Agent, and from the Agent to the Customer*****:** This is the most popular method foreign organizations use when entering the Ghanaian market. Most of the organizations deal in office equipment, machines, vehicles installations and industrial raw material. Their Ghanaian counterparts provide after-sales service, training and installation services on behalf of their principals.

- ***Producer* to an *Agent,* from the *Agent* to an *Industrial distributor* and from the *Industrial distributor* to the *Customer*:** A good example of organizations involved in this sort of channel is Mechanical Lloyd- an agent of Yokohama tires in the country which markets these tires through a wide network of industrial distributors.

2. METHODSFOR SOLVING TRANSPORTATION PROBLEMS

There are several methods for solving transportation problems. Two of such methods are the Stepping Stone and Lagrangian Relaxation based methods. These methods are variants of the Simplex Method. The methods use aninitial BFS computed from methods like the Northwest corner rule or Vogel's Approximation Method, and improve upon the initial basic feasible solution to obtain an optimal solution.

Definitions

Cell: It is a small compartment in the transportation tableau.

Circuit: A circuit is a sequence of cells (in the balanced transportation tableau) such that

(i) It starts and ends with the same cell.
(ii) Each cell in the sequence can be connected to the next member by a horizontal or vertical line in the tableau.

Allocation: The number of units of items transported from a source to a destination which is recorded in a cell in the transportation tableau.

Basic Variables: The variables in a basic solution whose values are obtained as the simultaneous solution of the system of equations that comprise the functional constraints.

Basic Feasible Solution: A solution is called a basic feasible solution if

i. It involves (m + n -1) cells with non-negative allocations.
ii. There are no circuits among the cells in the solution.

2.1 Finding Initial Basic Feasible Solution of Balanced Transportation Problems

***The Northwest Corner Rule*:** The North West corner rule is a method for computing an initial basic feasible solution of a transportation problem where the basic variables are selected from the North – West corner (i.e., succeeding top left corner) of the transportation tableau. Given a balanced transportation problem in a transportation tableau, the following steps have been provided by Wayne L. Winston (Ntaimo, 2005) as the rule.

- Begin in the upper left (or northwest) corner of the transportation tableau
- Set x_{11} as large as possible. Clearly $x_{11} = \min\{s_1, d_1\}$.
- If $x_{11} = s_1$, cross out row 1 of the transportation tableau; no more basic variables will come from row 1. Also set $d_1 = d_1 - s_1$.
- If $x_{11} = d_1$, cross out column 1 of the transportation tableau; no more basic variables will come from column 1. Also set $s_1 = s_1 - d_1$.
- If $x_{11} = s_1 = d_1$, cross out either row 1 or column 1 (but not both).
  - If you cross out row 1, set $d_1 = 0$
  - If you cross out column 1, set $s_1 = 0$.

Continue applying this procedure to the most northwest corner cell in the tableau that does not lie in the crossed-out row or column until you eventually reach a point where there is only one cell that can be assigned a value. Assign this cell a value equal to its row or column demand, and cross out the cell's row and column. A BFS has now been obtained.

Remark: In cases of Degeneracy, the solution obtained by the Northwest Corner method is not a basicfeasible solution because it has fewer than (m + n – 1) cells in the solution. This happens because at some point during the allocation, when a supply is used up, there is no cell with unfulfilled demand in the column. To resolve degeneracy a zero allocation is assigned to one of the unused cell. Although there is a great deal of flexibility in choosing the unused cell for thezero allocation, the general procedure, when using the northwest corner rule, is toassign it to a cell in such a way that it maintains an unbroken chain of allocated cells.

Example 1: Consider the balanced transportation problem in Table 2.3below, where the x_{ij} 's initially put to be $x_{ij} = 0$ or blank.

Table A balanced transportation problem

		Destination				Suppl
		1	2	3	4	
Source	1	6	5	7	9	40
	2	3	2	4	1	40
	3	7	3	9	5	25
Dema		30	20	35	20	

Applying the *Northwest Corner Rule*, we obtain the ordered allocations shown in Table 2.4below. The number of circled allocation is 6 (thatis 3 + 4 – 1 = 6) which gives the initial basic feasible solution. The arrows have been added to show the order in which the basic variables (allocations) were selected.

Table Initial BF solution from the Northwest Corner Rule

		Destination				Supply
		1	2	3	4	
Source	1	6 (30) →	5 (10) ↓	7	9	40
	2	3	2 (10) →	4 (30) ↓	1	40
	3	7	3	9 (5) →	5 (20)	25
Demand		30	20	35	20	

Hence the initial BFS is given by $\begin{bmatrix} x_{11} = 30, x_{12} = 10 \\ x_{22} = 10, x_{23} = 30 \\ x_{33} = 5, x_{34} = 20 \end{bmatrix}$

Cost Z $= \sum \sum c_{ij} x_{ij} = (30 \times 6) + (10 \times 5) + (10 \times 2) + (30 \times 4) + (5 \times 9) + (20 \times 5)$

$= 515$

Vogel's Approximation Method (VAM): Vogel's approximation method has been a popular criterion for many years. VAM usually yields a better initial solution than the other initial basic feasible solution methods(Mathirajan and Meenakshi, 2004). VAM is not quite as simple as the Northwest corner approach, but it facilitates a very good initial solution - as a matter of fact, one that is often the *optimal* solution. Vogel's approximation method tackles the problem of finding a good initial basic feasible solution by taking into account the costs associated with each route alternative. This is something that the northwest corner rule does not do. To apply the VAM, the steps below are followed:

1. For each row and each column of the transportation tableau, wefind the difference between the two lowest unit shipping costs. These numbers represent the difference between the distribution costs on the best route in the row or column and the second best least cost route in the row or column. It is also the opportunity cost.

2. We then identify the row or column with the greatest opportunity cost and assign the least of supply or demand capacities to the cell with the least cost of this row or column. Ties are broken arbitrarily.

3. We eliminate any row or column that has just been completely satisfied by the assignment just made and subtract the assignment from the supply or demand of row or column of the relevant assigned cell.
4. We re-compute the cost differences for the new transportation tableau, omitting rows or columns crossed out in the preceding step.

5. We then return to step 2 and repeat the steps until an initial feasible solution is obtained.

The method isillustrated by applying it to the balanced transportation problem in Table 2.3of section 2.3.1.1above; Table 2.5 shows the processes of obtaining solution.

Table Row and Column differences leading to elimination of column 4

		Destination 1	2	3	4	Supply	Row Difference
Source	1	6	5	7	9	40	1
	2	3	2	4	(1)	40-20	1
	3	7	3	9	5	25	2
Demand		30	20	35	(20)		
Column Difference		3	1	3	(4)		

Select $x_{24} = 20$

Eliminate column 4

Table Row and Column differences leading to elimination of column 2

		Destination			Supply	Row Difference
		1	2	3		
Source	1	6	5	7	40	1
	2	3	2	4	20	1
	3	7	3	9	25-20	4
Demand		30	20	35		
Column Difference		3	1	3		

Select $x_{32} = 20$

Eliminate column 2

Table Row and Column difference leading to elimination of row 2

		Destination		Supply	Row Difference
		1	3		
Source	1	6	7	40	1
	2	3	4	20	1
	3	7	9	5	2
Demand		30-20	35		
Column Difference		3	3		

Select $x_{21} = 20$

Eliminate row 2

Table Row and column differences leading to elimination of column 3

		Destination		Supply	Row Difference
		1	3		
Source	1	6	7	40-35	1
	3	7	9	5	2
Demand		10	35		
Column Difference		1	2		

Select $x_{13} = 35$

Eliminate column 3

Table Selection of column 1 for being the only column left

		Destination	Supply
		1	
Source	1	6	5
	3	7	5
Demand		10	

Hence, the initial BFS is given by
$$\begin{bmatrix} x_{11} = 5, x_{13} = 35 \\ x_{21} = 20, x_{24} = 20 \\ x_{31} = 5, x_{32} = 20, \end{bmatrix}$$
Select $x = 5, x = 5$

Cost,Z $= \sum\sum c_{ij}x_{ij} = (20\times1)+(20\times3)+(20\times3)+(35\times7)+(5\times7)+(5\times6)$
= 450

2.2 Method for Solving Transportation Problems to Optimality

***The Stepping Stone Method*:** This method determines the alternate cell with no allocation that would reduce cost if used. Considering the balanced transportation problem shown in Table 2.3, suppose that the BFS of this problem consists of (m + n – 1) non negative allocations (occupied)cells. Let the cells that are not in the BFS be known as *unoccupied cells*. The stepping Stone method uses the steps below to obtain an optimal solution to the transportation problem;

1. Test for optimality: For each of the unoccupied cells, form a circuit of horizontal and vertical lines,beginning with a plus (+) sign at the unoccupied cell. Thereafter place alternate minus (-) and plus (+) signs on each corner cell of the closed path traced, with the unoccupied cell being a corner cell and the other corners cells being occupied cells.

2. Using the unit cost of each cell, a closed path is formed for the unoccupied cell. We assign each unit cost by the relevant plus or minus. The total change in cost for the unoccupied cell that was used to form the circuit is computed. This change in cost is called *improvement index* of the unoccupied cell.

3. (i) If the improvement index of each unoccupied cell in the BFS is non negative, then the current BFS is optimal since any re-allocation increases the cost.

(iii) If there is at least one unoccupied cell with a negative improvement index, then a re-allocation to produce a new BFS with a lower cost is possible. Go to step 4.

4. Improvement to optimality.
   i. We find the unoccupied cell whose circuit produced the most negative improvement index.
   ii. Using the above circuit, we find the smallest allocation in the cells of the circuit with the " – " sign and denote this smallest allocation by*m*.Subtract *m* from the allocations in all the cells in the circuit with " – " sign and add to all the allocations in the cells in the circuit with " + " sign. This has the effect of satisfying the constraints on demand and supply in the transportation tableau.
   iii. Since the cell which carried the allocation *m* now has a zero allocation, it is deleted from the solution and is replaced by the cell in the circuit which was originally unoccupied and now has an allocation *m*.
   iv. The result of the re- allocation is a new basic feasible solution. The cost of this new basic feasible solution is *m*lessthan the cost of the previous BFS.
   v. Using the new BFS,go to step 1.

Example: Consider the balanced transportation problem in Table 2.3above.
From the *Northwest Corner Rule*, the initial basic feasible solution is shown circled in Table 2.6below and with the cost Z = 515.

Table Northwest Corner rule BFS

		Destination				Supply
		1	2	3	4	
Source	1	6 (30)	5 (10)	7	9	40
	2	3	2 (10)	4 (30)	1	40
	3	7	3	9 (5)	5 (20)	25
Demand		30	20	35	20	

The cost associated, Z =515.

First Iteration

Test for Optimality

The unoccupied cells are (1, 3), (1, 4), (2, 1), (2, 4), (3, 1) and (3, 2).

Computing improvement indices for theunoccupied cells:

For (1, 3):

The circuit is (1, 3) → (2, 3) → (2, 2) → (1, 2) → (1, 3)
+ - + - +

Improvement index = 7 – 4 + 2 – 5 = 0

For (1, 4):

The circuit is (1, 4) → (3, 4) → (3, 3) → (2, 3) → (2, 2) → (1, 2) → (1, 4)
+ - + - + - +

Improvement index = 9 – 5 + 9 – 4 + 2 – 5 = 6

For (2, 1):

The circuit is (2, 1) → (2, 2) → (1, 2) → (1, 1) → (2, 1)
+ - + - +

Improvement index = 3 – 2 + 5 – 6 = 0

For (2, 4):

The circuit is (2, 4) → (2, 3) → (3, 3) → (3, 4) → (2, 4)
+ - + - +

Improvement index = 1 – 4 + 9 – 5 = 1

For (3, 1):

The circuit is (3, 1) → (1, 1) → (1, 2) → (2, 2) → (2, 3) → (3, 3) → (3, 1)
+ - + - + - +

Improvement index = 7 – 6 + 5 – 2 + 4 – 9 = - 1

For (3, 2):

The circuit is (3, 2) → (3, 3) → (2, 3) → (2, 2) → (3, 2)
+ - + - +

Improvement index = 3 - 9 + 4 – 2 = - 4

<u>Improvement to optimality</u>

The unoccupied cell with the most negative improvement index is **(3, 2).** The least allocation to the cells in the circuit of (3, 2) with minus sign is 5. Subtracting this from the allocation of cells with the sign "-", in the circuit and adding it to the allocations in the cells in the circuit with the sign "+", we obtain the following new basic feasible solution as shown in Table 2.7.

Table 2.7: New BFS obtained from Stepping Stone Method

From \ To	1	2	3	4	Supply
1	6 (30)	5 (10)	7	9	40
2	3	2 (5)	4 (35)	1	40
3	7	3 (5)	9	5 (20)	25
Demand	30	20	35	20	

The new BFS is $x_{11} = 30, x_{12} = 10, x_{22} = 5, x_{23} = 35, x_{32} = 5$ and $x_{34} = 20$

Second Iteration

Test for Optimality

The unoccupied cells in the new solution are (1, 3), (1, 4), (2, 1), (2, 4), (3, 1), (3, 3). The improvement indices are shown in Table 2.8.

Table 2.8: Improvement indices of unoccupied cells in Table 2.7

Cell	(1, 3)	(1, 4)	(2, 1)	(2, 4)	(3, 1)	(3, 3)
Improvement Index	0	2	0	-3	3	4

Since there is an unoccupied cell with a negative improvement index, it follows that the current BFS is not optimal.

Improvement to optimality

The unoccupied cell with the most negative improvement index is (2, 4). The least allocation in the cells in its circuit with the sign "-"is 5. Subtracting it from the allocation in the other cell in the circuit with the sign "-" and adding to the allocations in the cells in the circuit with the sign "+", we obtain the new feasible solution in Table 2.9.

Table 2.9: Second BFS using Stepping Stone method

From \ To	1	2	3	4	Supply
1	6 (30)	5 (10)	7	9	40
2	3	2	4 (35)	1 (5)	40
3	7	3 (10)	9	5 (15)	25
Demand	30	20	35	20	

The new BFS is $x_{11} = 30, x_{12} = 10, x_{23} = 35, x_{24} = 5, x_{32} = 10$ and $x_{34} = 15.$

Third Iteration

Test for Optimality

The unoccupied cells in the current basic feasible solution are (1, 3), (1, 4), (2, 1), (2, 2), (3, 1) and (3, 3).The improvement indices are shown in Table 2.10.

Table 2.10: Improvement indices of unoccupied cells in Table 2.9

Cell	(1, 3)	(1, 4)	(2, 1)	(2, 2)	(3, 1)	(3, 3)
Improvement Index	*-3*	*2*	*3*	*3*	*3*	*1*

Since there is an unoccupied cell with a negative improvement index, it follows that the current basic feasible solution is not optimal. The unoccupied cell with the most negative improvement index is (1, 3). The least allocation to the cells in this circuit with the sign "-" is 10. . Subtracting it from the allocation in the other cell in the circuit with the sign "-" and adding to the allocations in the cells in the circuit with the sign "+", we obtain the new feasible solution in Table 2.11.

Table 2.11: Optimal solution from Stepping Stone method

From \ To	1	2	3	4	Supply
1	6 (30)	5	7 (10)	9	40
2	3	2	4 (25)	1 (15)	40
3	7	3 (20)	9	5 (5)	25
Demand	30	20	35	20	

The new BFS is $x_{11} = 30, x_{13} = 10, x_{23} = 25, x_{24} = 15, x_{32} = 20$ and $x_{34} = 5.$

Fourth Iteration

Test for optimality

The unoccupied cells in the current basic feasible solution are (1, 2), (1, 4), (2, 1), (2, 2), (3, 1) and (3, 3). The improvement indices are shown in Table 2.12.

Table 2.12: Improvement indices of unoccupied cells in Table 2.10

Cell	(1, 2)	(1, 4)	(2 , 1)	(2, 2)	(3, 1)	(3, 3)
Improvement Index	3	5	0	3	0	1

Since there is no unoccupied cell with a negative improvement index, it follows that the current basic feasible solution is optimal. The optimal solution is given by

$x_{11} = 30, x_{13} = 10, x_{23} = 25, x_{24} = 15, x_{32} = 20$ and $x_{34} = 5$ with the minimum cost of 450.

***Lagrangian Relaxation Based Methods*:** One of the most computationally useful ideas of the 1970s is the observation that many hard problems can be viewed as easy problems complicated by a relatively small set of side constraints. Making the side constraints dual produces a Lagrange problem that is easy to solve, and whose optimalvalue is a lower bound (for minimization problems) on the optimal value of the original problem. The "birth" of Lagrangian approach as it exists today occurred in 1970when Held and Karp (1970, 1971) used a Lagrangian problem based on minimum spanning trees to devise a dramatically successful algorithm for the traveling salesman problem. Motivated by Held and Karp's success, Lagrange methods were applied in the early 1970s to scheduling problems (Fisher, 1973). Lagrangian methods had gained considerable currency by 1974 when Geoffrion (1974) coined the perfect name for this approach – "Lagrangian Relaxation".

Equality Constraints for Lagrangian Function: Given the problem $P1: \quad \text{minimize } f(x)$
$\text{subject to } g(x) = b, \quad x \in X$.

The Lagrangian function is defined to be

$$L(x,\lambda) = f(x) + \lambda^T (b - g(x)).$$

The components $\lambda = (\lambda_1, ..., \lambda_m)$ are known as the Lagrange multipliers.

***Inequality Constraints and Complementary Slackness*:** When the functional constraints in the problem P1 are in inequality form the problem becomes

$$P2: \text{ minimize } f(x)$$

$$\text{subject to } g(x) \leq b, \quad x \in X.$$

It may be expressed in the previous form with equality constraints using slack variables as

$$\text{P3:minimize } f(x),$$

$$\text{subject to } g(x) + z = b, x \in X \text{ and } z \geq 0.$$

The Lagrangian now becomes

$L(x,z,\lambda) = f(x) + \lambda^T (b - g(x) - z)$, and it must be minimized over $x \in X$ and $z \geq 0$.

Consider the term in the Lagrangian involving $-\lambda_i z_i$; if $\lambda_i > 0$ then letting z_i become arbitrarily large shows that this term can be made to approach $-\infty$ which implies that $\inf_{x \in X, x \geq 0} L(x,z,\lambda) = -\infty$. Thus, for a finite minimum of the Lagrangian we require that $\lambda_i \leq 0$, in which case the minimum of the term $-\lambda_i z_i$ is 0, since we could take $z_i = 0$ Thus, with the inequality constraints in the problem, minimizing the Lagrangian always leads to sign conditions on the Lagrange multipliers, in this case $\lambda \leq 0$. There is also a joint condition on the Lagrange multipliers and the slack variables in that $\lambda_i z_i = 0 \quad \text{for each } i = 1,...,m, \text{ or equivalently, } \lambda^T z = 0.$

This condition is known as a complementary slackness condition; at least one of the variables λ_i and z_i must be zero (at the optimum solution) for each i.

***Lagrange Multipliers and the Transportation Problem*:** A classical optimization problem is the transportation problem in which there are *m* sources of supply of a particular good $\{S_1, ..., S_m\}$, with amounts $\{s_1, ..., s_m\}$ available, and *n* destinations $\{D_1, ..., D_n\}$ at which there are demands $\{d_1, ..., d_n\}$, respectively for the good. For each pair $\{S_i, D_j\}$, there is a cost c_{ij} per unit for shipping from S_i to D_j.

***Assumption*:** $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$,that is, total supply equals total demand.

Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-35, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

The objective is to satisfy the demand from the supplies with the minimal transportation cost. Let x_{ij} denote the flow from S_i to D_j.

The transportation problem is the linear programming problem formulated as

$$\text{Minimize} \quad Z=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \text{.....................(1)}$$

$$\text{subject to} \quad \sum_{j=1}^{n} x_{ij} = s_i \quad (i=1,2,...,m) \text{ ....(2)} \qquad \text{(Supply constraints)}$$

$$\sum_{i=1}^{m} x_{ij} = d_j \quad (j=1,2,...,n) \text{ ....(3)}$$

$$x_{ij} \geq 0 \quad (i=1,2,...,m;\ j=1,2,...,n) \text{ ....(4)} \quad \text{(Demand constraints)}$$

Let λ_i and v_j be the Lagrange multipliers.

The Lagrangian for the balanced transportation problem is

$$L(x,\lambda,v)=\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}+\sum_{i=1}^{m}\lambda_i\left(s_i-\sum_{j=1}^{n}x_{ij}\right)+\sum_{j=1}^{n}v_j\left(d_j-\sum_{i=1}^{m}x_{ij}\right)$$

$$=\sum_{i=1}^{m}\sum_{j}^{n}(c_{ij}-\lambda_i-v_j)x_{ij}+\sum_{i=1}^{m}\lambda_i s_i+\sum_{j=1}^{n}v_j d_j$$

The minimum of the Lagrangian over $x_{ij} \geq 0$ will be finite provided:

$c_{ij}-\lambda_i-v_j \geq 0,$ for each i, j (dual feasibility)

and at the optimum

$(c_{ij}-\lambda_i-v_j)x_{ij}=0,$ for each $i, j.$ (complementary slackness)

The steps for Lagrangian procedure for solving balanced transportation problems are then indicated as follows;

1. *Initial assignment*. We start the algorithm by choosing an initial basic feasible solution (BFS) by the Northwest method.
2. *Assign the Lagrangian multipliers.*Next, we choose the values for the Lagrange multipliers $(\lambda_i),(v_j)$ so that $c_{ij}-\lambda_i-v_j=0$ for the basic cells; this ensures that the complementary slackness holds. Since only the sum λ_i+v_j enter into all the calculations one of these multipliers may be chosen arbitrarily, $\lambda_1=0$, say.
3. *Test for optimality*. We identify the non-basic cells for which $c_{ij}-\lambda_i-v_j<0$; if all cells have $c_{ij}-\lambda_i-v_j \geq 0$ then the current solution is optimal. Otherwise go to step 4.
4. *Pivoting*. Choose the non-basic cell with the most negative value of $c_{ij}-\lambda_i-v_j$ (Pivote cell). Put an amount $\epsilon>0$ units of flow into the pivot cell. At the same time, add or subtract from the basic cells to maintain feasibility. Now choose the largest ϵ possible such that the flow is feasible.
5. The algorithm now returns to step 2 with this flow as the basic feasible flow.

Let us apply these steps to the initial BFS obtained from the Northwest method in Table 2.5with the total cost of flow of 515.

First Iteration

Step 2: We choosevalues for the Lagrange multipliers $(\lambda_i),(v_j)$ so that $c_{ij}-\lambda_i-v_j=0$ for the basic cells. We obtain the following equations;

$6-\lambda_1-v_1=0$

$5-\lambda_1-v_2=0$

$2-\lambda_2-v_2=0$

$4-\lambda_2-v_3=0$

$9-\lambda_3-v_3=0$

$5-\lambda_3-v_4=0$

Letting $\lambda_1=0,$ we obtain $v_1=6, v_2=5, \lambda_2=-3, v_3=7, \lambda_3=2$ and $v_4=3$

The values for λ_i and v_j obtained for the basic cells are as shown in Table 2.13.

Table Lagrange multipliers of basic cells

λ_i \ v_j	6	5	7	3
0	[6] 30	[5] 10	[7]	[9]
-3	[3]	[2] 10	[4] 30	[1]
2	[7]	[3]	[9] 5	[5] 20

Step 3: Check for optimality: For each non-basic (unoccupied) cell we compute $c_{ij}-\lambda_i-v_j$ and identify those with $c_{ij}-\lambda_i-v_j<0$. The non-basic cells and their $c_{ij}-\lambda_i-v_j$ values are shown in Table 2.14below.

Table Non-basic cells and their c_{ij} - λ_i - v_j values

Non-basic cell	**Value of** $c_{ij}-\lambda_i-v_j$
(1,3)	7 - 0- 7 = 0
(1,4)	9 – 0 – 3 = 6
(2,1)	3 – (- 3) – 6 = 0
(2,4)	1 – (- 3) – 3 = 1
(3,1)	7 – 2 – 6 = - 1
(3,2)	3 – 2 – 5 = - 4

Since some of the $c_{ij} - \lambda_i - v_j$ values are negative, it means that the solution is not optimal and therefore the pivot operation must occur.

Step 4: The nonbasic cell with the most negative $c_{ij} - \lambda_i - v_j$ value is (3,2). We increase the solution in this cell by ε and form a loop as shown in Table 2.15(a)

Table 2.15(a): New allocation with ε adjustment for first iteration

[6] 30	[5] 10	[7]	[9]
[3]	[2] 10 - ε	[4] 30 + ε	[1]
[7]	[3] ε	[9] 5 - ε	[5] 20

We then increase ε until the allocation in one of the basic cells becomes zero; in this case when ε = 5, and this gives a new basic feasible solution as shown in Table 2.15(b)below.

Table 2.15(b): New BFS for First Iteration using Lagrange Multipliers

[6] 30	[5] 10	[7]	[9]
[3]	[2] 5	[4] 35	[1]
[7]	[3] 5	[9]	[5] 20

The solution is $x_{11} = 30, x_{12} = 10, x_{22} = 5, x_{23} = 35, x_{32} = 5$ and $x_{34} = 20.$

Total cost of flow, $Z = (6 \times 30) + (5 \times 10) + (2 \times 5) + (4 \times 35) + (3 \times 5) + (5 \times 20) = 490$

Second Iteration

The algorithm returns to step 2 with the current solution as the basic feasible solution. Following the steps in the first iteration we get λ = [0, -3, -2] and v = [6, 5, 7, 7].An (X) has been placed in the non-basic cell for which $c_{ij} - \lambda_i - v_j < 0$. The numerical difference ($c_{ij} - \lambda_i - v_j$) of this non basic cell is - 3.Table 2.16(a) shows the new allocation with ε adjustment.

Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-35, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

Table 2.16(a): New allocation with ϵ adjustment for Second Iteration

λ_i \ v_j	6	5	7	7
0	(6) 30	(5) 10	(7)	(9)
-3	(3)	(2) 5 - ε	(4) 35	(1) ε X (- 3)
-2	(7)	(3) 5 + ε	(9)	(5) 20 - ε

Table 2.16(b)below shows the new BFS with total cost 485.

Table 2.16(b): New BFS for second iteration

(6) 30	(5) 10	(7)	(9)
(3)	(2)	(4) 35	(1) 5
(7)	(3) 10	(9)	(5) 15

Total cost = 485

The new BFS is $x_{11} = 30, x_{12} = 10, x_{23} = 35, x_{24} = 5, x_{32} = 10$ and $x_{34} = 15.$

Third Iteration

The algorithm returns to step 2 with the current solution as the basic feasible solution. Following the steps in the first iteration we get λ = [0, -6, -2] and v = [6, 5, 10, 7]. An (X) has been placed in the non-basic cell for which $c_{ij} - \lambda_i - v_j < 0$. The numerical difference ($c_{ij} - \lambda_i - v_j$) of the cell (i.e., (1, 3)) is – 3.Table 2.17(a)shows the new allocation with ϵ adjustment.

Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-35, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

Table 2.17(a): New allocation with ϵ adjustment for ThirdIteration

λ_i \ v_j	6	5	10	7
0	[6] 30	[5] 10 - ε	[7] ε X (-3)	[9]
-6	[3]	[2]	[4] 35 - ε	[1] 5 + ε
-2	[7]	[3] 10 + ϵ	[9]	[5] 15 - ε

The table below shows the new BFS with the total cost 450.

Table 2.16(b): New BFS for Third Iteration

[6] 30	[5]	[7] 10	[9]
[3]	[2]	[4] 25	[1] 15
[7]	[3] 20	[9]	[5] 5

Total cost = 450,

Since the all the non-basic cells in the above tableau satisfy the dual feasibility condition (i.e., $c_{ij} - \lambda_i - v_j \geq 0$), it means that the current basic feasible solution $x_{11} = 30, x_{13} = 10, x_{23} = 25\ x_{32} = 20, x_{34} = 5$ is optimal. The cost associated Z = 450.

3. DATA COLLECTION

Data was collected from the factory site of Latex Foam Company Limited, Kumasi. The company operates eight models of trucks. The list below gives the types of trucks;

i. KIA truck (K)
ii. TATA truck (T)
iii. Renault (articulator) truck (RA)
iv. TATA (articulator) truck (TA)
v. Benz (articulator) truck (BA)
vi. DAF (articulator) truck (DA)
vii. DAF (cargo) truck (DC)

viii. Benz (cargo) truck (BC)

There are three KIA trucks (K1, K2, and K3), four TATA trucks (T1, T2, T3, and T4) and two Benz articulator trucks (BA1, BA2). The rest are single trucks. The trucks ply routes along which they serve various customers with the final destinations mostly being District Capitals. The final destinations are used as the starting point for supply. The list of these final destinations is given below; Sefwi Juaboso (D1), Asankraguaa (D2),Yendi (D3), Assin Fosu (D4), Kintampo (D5), Kwame Danso (D6), Bogoso (D7), Osei Kojokrom (D8), Bawku (D9), Drobo (D10), Goaso (D11), Yeji (D12), Lawra (D13), Juaso/Obogu (D14), Nkawkaw (D15), Gushiegu (D16). The cost of a trip from the factory shed in Kumasi to a destination is measured in gallons of diesel used. Table 3.1 shows the cost of a trip when the trucks are assigned to the various destinations.

Table 3.1:Typesof Trucks and quantity of diesel (in gallons) used per trip

Type of truck	Sefwi Juabeso	Asankraguaa	Yendi	Assin Fosu	Kintampo	Kwame Danso	Bogoso	Osei Kojokrom	Bawku	Drobo	Goaso	Yeji	Lawra	Obogu	Nkawkaw	Gushiegu
K	28	22	40	12	16	25	20	35	-	16	12	20	-	9	10	-
T	29	23	43	13	18	24	22	37	-	18	13	21	-	10	11	-
RA	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
TA	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
BA	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
DA	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
DC	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
BC	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

The KIA and TATA trucks are restricted from going far places and therefore data on the fuel consumptionrates for these trucks were not available for places like Gushiegu, Lawra and Bawku. The M found in Table 3.2 shows that the trucks involved are prohibited from going to those places. The cost (c_{ij}) of Table 3.2 below was obtained from Table 3.1 for all the fourteen trucks.

C_{ij} representthe cost of assigning vehicle $i \in V$ to route $j \in D$ where V1 = K1,V2 = K2,V3 = K3,

V4 = T1,V5 = T2, V6 = T3,V7 = T4,V8 = RA,V9 = TA,V10 = BA1, V11= BA2,V12 = DA,

V13 = DC and V14 = BC.

Table 3.2: Cost matrix obtained from Table 3.0

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

4. PROBLEM FORMULATION

The problem is to find the minimum total cost in assigning each truck to a distinct destination. The problem is formulated as an assignment problem with the assumption that a truck is assigned to only one route on which there may be more than one depot. The mathematical notation and formulation are as follows.

Let

$C_{ij} =$ cost coefficients(number of gallons of diesel) of assigning truck type i from factory to route j.

$V =$ Set of all vehicles.

$D =$ Set of all destinations

m = Total number of trucks

n = number of routes to the final destinations

The Boolean variables, X_{ij}, representing assignment realization are defined by

$$X_{ij} = \begin{cases} 1 \text{ If truck type } i \text{ is assigned from factory to route } j \\ 0 \text{ otherwise} \end{cases}$$

The objective function (Z) can be written as

$$\text{Minimize } Z = \sum_{i \in v}^{m} \sum_{j \in u}^{n} c_{ij} x_{ij} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (1),$$

subject to

$$\sum_{j}^{n} x_{ij} = 1, \text{ for all } i \in V \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$\sum_{i \in v}^{m} x_{ij} = 1, \text{ for all } j \in U \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

$x_{ij} = 0$ or $x_{ij} = 1$............(4)

The objective function (1) is to minimize the total cost in terms of number of gallons of diesel used for the assignments. Constraint (2) requires that each truck is assigned exactly one route to a destination.
Constraint (3) requires that every route to a destination is assigned to only one truck.

Constraint (4) requires that a particular truck i is assigned to a distinct destination j. (i.e. $x_{ij} = 1$) or otherwise ($x_{ij} = 0$)

For efficient assignments of these trucks, the cost matrix of Table 3.2 must be a square one. In order to obtain a square cost matrix, two trucks from the six brands (in terms of fuel consumption), that is KIA truck, TATA truck, Renault or TATA or DAF (articulator) truck, Benz (articulator) truck, DAF (cargo) truck and Benz (cargo) truck, were selected and added in turn to the existing fourteen vehicles to obtain a 16×16 matrix. In all, twenty-onecost matrices were obtained. Tables 3.3 - 3.23 show these matrices.

Table 3.3: Cost matrix for adding two KIA trucks

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M

Table 3.4: Cost matrix for adding a KIA truck and a TATA truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98

V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M

Table 3.5: Cost matrix for adding a KIA truck and a Renault, TATA or DAT (articulator) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98

Table 3.6: Cost matrix for adding a KIA truck and a Benz (articulator) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96

Table 3.7 Cost matrix for adding a KIA truck and a DAF (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

Table 3.8: Cost matrix for adding a KIA truck and a Benz (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

Table 3.9: Cost matrix for adding two TATA trucks

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M

Table 3.10: Cost matrix for adding a TATA truck and a Renault, TATA or DAF (arti.) truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98

Table 3.11: Cost matrix for adding a TATA truck and a Benz (articulator) truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96

Table 3.12: Cost matrix for adding a TATA and a DAF (cargo) truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V1	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

Table 3.13: Cost matrix for adding a TATA and a Benz (cargo) truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V13	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

Table 3.14: Cost matrix for adding two of Renault, TATA or DAF (articulator) trucks

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98

Table 3.15: Cost matrix for adding Renault, TATA or DAT (arti.) truck and a Benz (arti.) truck

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M

V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96

Table 3.16: Cost matrix for adding Renault, TATA or DAT (arti.) truck and a DAF (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	28	38	55	83	25	20	30	75	16	17	76

Table 3.17: Cost matrix for adding Renault, TATA or DAT (arti.) truck and a Benz (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98

V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V8	55	50	85	29	35	55	50	60	110	35	22	45	115	25	19	98
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

Table 3.18: Cost matrix for adding two Benz (articulator) trucks

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V10	54	49	83	28	34	28	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	28	48	57	108	34	28	43	113	24	18	96

Table 3.19: Cost matrix for adding a Benz (arti.) truck and a DAF (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

Table 3.20: Cost matrix for adding a Benz (arti.) truck and a Benz (cargo) truck

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

Table 3.21: Cost matrix for adding two DAF (cargo) trucks

	D1	**D2**	**D3**	**D4**	**D5**	**D6**	**D7**	**D8**	**D9**	**D10**	**D11**	**D12**	**D13**	**D14**	**D15**	**D16**
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76

Table 3.22: Cost matrix for adding a DAF (cargo) and a Benz (cargo) trucks

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

Table 3.23: Cost matrix for adding two Benz (cargo) trucks

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15	D16
V1	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V2	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V3	28	22	40	12	16	25	20	35	M	16	12	20	M	9	10	M
V4	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V5	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V6	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V7	29	23	43	13	18	26	22	37	M	18	13	21	M	10	11	M
V8	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V9	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V10	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V11	54	49	83	28	34	53	48	57	108	34	28	43	113	24	18	96
V12	55	50	85	29	35	55	50	60	110	35	29	45	115	25	19	98
V13	40	42	73	20	28	30	38	55	83	25	20	30	75	16	17	76
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74
V14	39	41	72	19	27	28	37	53	81	23	19	29	73	15	16	74

6. RESULTS AND DISCUSSION

A Mat Lab code,written by Buehren (2008), was used to implement the Munkres algorithm on a Pentium (IV) computer of processor speed 2.60GHz using the data of Tables 3.3 – 3.23. The output of the program for each of the twenty-one cost matrices is given in Table 3.24 below.

Table 3.24: Results obtained from Cost Matrices using Mat lab code

Table Number	*Assignments($x_{ij}=1$ values)*	*Cost* $Z=\sum_{i=1}^{16}\sum_{j=1}^{16}c_{ij}x_{ij}$
3.3	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,2}=1, x_{5,12}=1, x_{6,4}=1, x_{7,1}=1, x_{8,14}=1$ $x_{9,6}=1, x_{10,11}=1, x_{11,16}=1, x_{12,15}=1, x_{13,13}=1, x_{14,9}=1, x_{15,5}=1, x_{16,7}=1$	572
3.4	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,7}=1, x_{5,1}=1, x_{6,12}=1, x_{7,4}=1, x_{8,15}=1$ $x_{9,11}=1, x_{10,16}=1, x_{11,6}=1, x_{12,14}=1, x_{13,9}=1, x_{14,13}=1, x_{15,5}=1, x_{16,2}=1$	574
3.5	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,4}=1$ $x_{9,14}=1, x_{10,15}=1, x_{11,16}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,5}=1, x_{16,6}=1$	590
3.6	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,4}=1$ $x_{9,14}=1, x_{10,15}=1, x_{11,6}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,5}=1, x_{16,16}=1$	589
3.7	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,13}=1, x_{14,9}=1, x_{15,5}=1, x_{16,16}=1$	569
3.8	$x_{1,10}=1, x_{2,8}=1, x_{3,7}=1, x_{4,5}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,15}=1$ $x_{9,6}=1, x_{10,4}=1, x_{11,11}=1, x_{12,14}=1, x_{13,9}=1, x_{14,16}=1, x_{15,3}=1, x_{16,13}=1$	567
3.9	$x_{1,8}=1, x_{2,3}=1, x_{3,5}=1, x_{4,2}=1, x_{5,1}=1, x_{6,10}=1, x_{7,4}=1, x_{8,11}=1$ $x_{9,6}=1, x_{10,14}=1, x_{11,16}=1, x_{12,15}=1, x_{13,9}=1, x_{14,13}=1, x_{15,7}=1, x_{16,12}=1$	576
3.10	$x_{1,10}=1, x_{2,3}=1, x_{3,5}=1, x_{4,12}=1, x_{5,7}=1, x_{6,8}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,16}=1, x_{12,15}=1, x_{13,9}=1, x_{14,13}=1, x_{15,2}=1, x_{16,6}=1$	592
3.11	$x_{1,10}=1, x_{2,3}=1, x_{3,5}=1, x_{4,12}=1, x_{5,7}=1, x_{6,8}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,9}=1, x_{14,13}=1, x_{15,2}=1, x_{16,16}=1$	591

Scholarly Journal of Mathematics & Science (https://damaacademia.com/sjms/)
Volume 1, Issue 3, pp.01-35, March, 2019
Published by: Dama Academic Scholarly & Scientific Research Society (www.damaacademia.com)

Continuation of Table 3.24

Table Number	*Assignments(* $x_{ij}=1$ *values)*	*Cost Z* $=\sum_{i=1}^{16}\sum_{j=1}^{16} c_{ij}x_{ij}$
3.12	$x_{1,10}=1, x_{2,3}=1, x_{3,5}=1, x_{4,12}=1, x_{5,7}=1, x_{6,8}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,13}=1, x_{14,9}=1, x_{15,2}=1, x_{16,16}=1$	571
3.13	$x_{1,10}=1, x_{2,3}=1, x_{3,5}=1, x_{4,12}=1, x_{5,7}=1, x_{6,8}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,9}=1, x_{14,16}=1, x_{15,2}=1, x_{16,13}=1$	569
3.14	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,2}=1, x_{5,7}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,4}=1, x_{10,15}=1, x_{11,16}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,14}=1, x_{16,6}=1$	609
3.15	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,2}=1, x_{5,7}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,6}=1, x_{10,15}=1, x_{11,4}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,14}=1, x_{16,16}=1$	608
3.16	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,16}=1, x_{14,9}=1, x_{15,11}=1, x_{16,13}=1$	588
3.17	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,6}=1, x_{12,11}=1, x_{13,9}=1, x_{14,16}=1, x_{15,15}=1, x_{16,13}=1$	586
3.18	$x_{1,8}=1, x_{2,3}=1, x_{3,10}=1, x_{4,2}=1, x_{5,7}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,6}=1, x_{10,15}=1, x_{11,16}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,14}=1, x_{16,4}=1$	607
3.19	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,16}=1, x_{14,9}=1, x_{15,11}=1, x_{16,13}=1$	587
3.20	$x_{1,10}=1, x_{2,3}=1, x_{3,8}=1, x_{4,7}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,5}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,6}=1, x_{12,11}=1, x_{13,9}=1, x_{14,16}=1, x_{15,15}=1, x_{16,13}=1$	584
3.21	$x_{1,7}=1, x_{2,3}=1, x_{3,8}=1, x_{4,5}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,15}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,6}=1, x_{12,11}=1, x_{13,13}=1, x_{14,10}=1, x_{15,16}=1, x_{16,9}=1$	577
3.22	$x_{1,7}=1, x_{2,3}=1, x_{3,8}=1, x_{4,5}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,11}=1$ $x_{9,4}=1, x_{10,14}=1, x_{11,15}=1, x_{12,6}=1, x_{13,9}=1, x_{14,13}=1, x_{15,16}=1, x_{16,10}=1$	576
3.23	$x_{1,7}=1, x_{2,3}=1, x_{3,8}=1, x_{4,5}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,15}=1$ $x_{9,6}=1, x_{10,14}=1, x_{11,4}=1, x_{12,11}=1, x_{13,9}=1, x_{14,13}=1, x_{15,10}=1, x_{16,16}=1$	574

From the Table 3.23 of results, table number 3.8 gives the smallest Z value (567). Hence the assignment $x_{1,10}=1, x_{2,8}=1, x_{3,7}=1, x_{4,5}=1, x_{5,2}=1, x_{6,12}=1, x_{7,1}=1, x_{8,15}=1 x_{9,6}=1, x_{10,4}=1, x_{11,11}=1, x_{12,14}=1$ $, x_{13,9}=1, x_{14,16}=1, x_{15,3}=1, x_{16,13}=1$ is optimal.

Table 3.24 below shows the above assignment of the trucks to the respective destinations.

Table 3.25: Optimal Assignment of Trucks

Type of Truck	*Route to be assigned/Final destination*
KIA (4)	Drobo, Oseikojokrom, Bogoso and Yendi
TATA (4)	Kintampo, Asankraguaa, Yeji and Sefwi Juabeso
Renault (art.)	Nkawkaw
TATA (art.)	Kwame Danso
DAF (art.)	Obugu
Benz (art.) (2)	Assin Fosu and Goaso
DAF (Cargo)	Bawku
Benz (Cargo) (2)	Lawra and Gushiegu

References

Amponsah, S.K. (2006). *Optimization Techniques* 1(MATH 507), University Printing Press, KNUST.

Andras, F. (2004). *On Kuhn's Hungarian Method-A tribute from Hungary* EGRES Technical Report No 2004-14

Anshuman, S. and Rudrajit, T. (2007). *Solving the Assignment Problem Using GeneticAlgorithm and Simulated Annealing*. IAENG International Journal of Applied Mathematics, 36:1

Beaujon G.J. and Turnquist M.A. (1991). *A Model for Fleet Sizing andVehicle Allocation*: *Transportation Science*, **25** (1), pp. 19-45

Bhatia, H.L., Swarup, K. and Puri, M.C. (1975). *A Procedure for Time MinimizingTransportation Problem*. http://w.w.w.new.dli.ernet.in/rawdataupload/upload/insa/INSA_2/200(*Accessed on* 24/05/2009)

Bradley, S. P., Hax, A.C. and Magnanti T.L. (1977). *Applied Mathematical Programming*,Addison-Wesley Publishing Company, San Francisco

Buehren, M. (2008). *Functions for Rectangular Assignment Problem*

http://www.mathworks.com/matlabcentral/fileexchange/6543 (*Accessed on* 28/02/2009)

Clark, G. and Van Der Werf, Y. (1998). *Work in Progress? TheIndustrious Revolution*.*The Journal of Economic History***58**; 3, pp 830-843

Clarke, C. and Wright, J.W. (1964). *Scheduling of Vehicles from Central Depot to a Numberof Delivery Points*. Operations Research, 12; pp 568-581

Cooke, W.P. (1985).*Quantitative Methods for Management Decision*. McGraw-Hill, New York.

Dantzig, G.B. and Ramser, J.H. (1959). *The Truck Dispatching Problem*, Management Science; pp 80-91

David, R A., Dennis, J.S. and Thomas, A. W. (1988).*An Introduction to Management Science (5th Edition)*: West Publishing Company, San Francisco.

De Vries (1994).*The Industrial Revolution and Industrious Revolution*: *Thejournal of Economic History* 54: (2) 249-270

Dipankar D., German H., Deon G., Pavan K. V, Aishwarya K., Ramjee Y. (2009). *A Comparison of Multi-objective Evolutionary Algorithms with Informed Initialization and Kuhn- Munkres Algorithm for the Sailor Assignment Problem,* Dept. of Computer Science, University of MemphisMemphis, TN 38152. (*Accessed on* 24/6/2009)

Fisher, M.L. (1973). *Optimal Solution of Scheduling Problems using LagrangeMultipliers: Part I*. Operations Research: Vol. 21, pp 1114-1127.

Ford, L.R. and Fulkerson,D.R. (1962).*Flow in Networks*, Princeton Univ. Press, Princeton NJ.

Geoffrion, A.M. (1974). *Lagrangian Relaxation and its Uses in Integer Programming*, Mathematics Programming Study: Vol. 2, pp 82-114

Green L.V. and Guha, D. (1995). *On the Efficiency of Imbalance in Multi-FacilityMulti-Server Service Systems*:*Management Science*, **41**, pp. 179-187

Hammer, P.L. (1969). *Time Minimizing Transportation Problems*, Naval Research Quarterly 16; pp 345-357

Hartvigsen, D., Wei, J.C. and Czuchlewski, R. (1999) "*conference paper-reviewer assignment problem, The"*Decision Sciences.

http://www.findarticles.com/p/articles/mi_qa3713/is_199907 (*Accessed on* 24/10/2008)

Hausah, R. (2009). *The Optimal Assignment Problem*
http://www.maths.qmul.ac.uk/~bill/MAS210/ch6.pdf (*Accessed on* 28/02/2009)

Held, M. and Karp, R. M. (1970).*The Travelling Salesman Problem and Minimum SpanningTrees: Part I*. Operations Research: Vol. **18**, pp 1138-1162

Held, M. and Karp, R.M. (1971).*The Travelling Salesman Problem and Minimum SpanningTrees: Part II*. Mathematical Programming: Vol. 1, pp 6-25

Hiller, F.S. and Lieberman,G. J. (2005). *Introduction to Operations Research*,McGraw-Hill, New York

Hitchcock, F.L. (1941). *The Distribution of a Product from Several Sources to Numerous Locations*, J. Math Phys.20, pp 224-230

Jardine, L. (1996*). Conspicuous Consumption Worldly Goods: A New History of theRenaissance*. Norton, New York.

Joubert, J.W. (2007). *The Vehicle Routing problem: Origins and Variants*, University of Pretoria edt., pp 4

Kim, S. (2000). *Markets and Multiunit Firms from an American HistoricalPerspective*

http://soks.wustl.edu/mumm.p.d.f (*Accessed on* 20/05/2009)

Kim, S. (1995). *Expansion of Markets and the Geographic Distribution of Economic Activities*: *Trends in U.S. Regional Manufacturing Structure, 1860-1987*, Quarterly Journal of Economics 110, 4: 881-908.

Koopman, T.C. (1947). *Optimum Utilization of Transportation System*, Proc. Intern. Statis. Conf., Washington, D.C

Löbel, A. (1998). *Vehicle Scheduling in Public Transit and LagrangianPricing*,*ManagementScience*, 44(12), pp. 1637-1649

Lotfi, V. and Pegels, C. (1989). *Decision Support Systems for Management Science.*Operations Research, Irwin, Homewood

Mahmoud, T.A. (1996). *A -Level Business Management* (4th edition), Folie Books Ent., Tema.

Mathirajan, M. and Meenakshi, B. (2004). *Experimental Analysis of some Variants ofVogel's Approximation Method*: Journal of Operations Research. Vol.21 (4), pp 449

Michael, Z.S., and Powell, W.B. (2003). *The Dynamic Assignment Problem*. Transportation Science, Vol. 38, 4, pp 399-419.

Ntaimo, L. (2005).*Transportation and Assignment Problems*(4th ed.) http://ise.tamu.ed/INEN420/INEN420_2005.Spring/SLIDES/chapter%207.pdf (*Accessed on* 05/08/2009)

Toth, P. and Vigo, D. (1997). Heuristic Algorithms for the Handicapped Persons Transportation Problem, Transportation Science, Vol. 31, No. 1, pp 60-71

Ross, E (1993). *Love and Toil Motherhood in Outcast London, 1870-1918*. Oxford University Press, New York

Rushmeier, R.A. and Kantogiorgis, S.A. (1997).*Advances in theOptimization of Airline Fleet Assignment: Transportation Science*, **31** (2), pp. 159-169.

Sharma, J.K. and Swarup, K. (1978). *Time Minimization in Transportation Problems*, N.Z.O.R 6; pp 75-88

Sokoloff, K. L. (1988). "Inventive Activity in Early Industrial America: Evidence from Patent Records 1790-1846," *Journal of Economic History* 48: 813-849

Sokoloff, K. L. (1984a). *Investment in Fixed and Working Capital during Early Industrialization: Evidence from U.S. Manufacturing Firms*, Journal of EconomicHistory44: 545-556.

Sokoloff, K.L. (1984b). *Was the Transition from the Artisanal Shop to the Non-Mechanized Factory Associated with Gains in Efficiency? Evidence from the U.S.Manufacturing Censuses of 1820 and 1850,*Explorations in Economic History21: 351-382.

Szwarc, W. (1964). *The Transportation Problem with Stochastic Demand*, ManagementScience, 11; pp 33-50.

Whitt, W. (1992). *Understanding the Efficiency of Multi-Server Service Systems*. *Management Science*, 38, pp. 708-723

Wiesner-Hanks, M.E. (2006). *Early Modern Europe 1450-1789*, pp 202-205,418-419

Wikipedia. (2009). *Industrial Revolution*http://en.wikipedia.org/wiki/Industrial_revolution(*Accessedon* 16/02/2009)

Williams, A.C. (1963*). A Stochastic Transportation Problem*, Operations Research, 11; pp 759-770

Wilson, D. (1972). *An a Priori Bounded Model for Transportation Problems with Stochastic Demand and Integer Solution*. AIIE Trans., 3; pp 186-193

Wilson, D. (1973). *Tighter Bounds for Stochastic Transportation Models*. AIIE Trans., 5; pp 180-185

Wilson, D. (1975). *A Mean Cost Approximation for Transportation Problems withStochastic Demand*, Naval Research Quarterly; 22

Woeginger,G.J, Burkard, R. and Cela, E. (1995).*A minimax assignment problem intreelikecommunication networks*,European Journal of Operational Research http://www.faqs.org/abstracts/Business-international/A-minimax-assignment-problem-in-treelike-communication-networks.html#ixzz0Sg8mXbSH (*Accessed on* 05/08/2009)

Xu, J. and Ye, X. (2008).A fuzzy Vehicle Routing Assignment Model, World Journal of Modeling and Simulation, Vol. 4; 2, pp 257-268

Zeleny, M. (1982). *Multiple Criteria Decision Making*, McGraw-Hill, New York.

Ziarati, K., Soumis ,F., Desrosiers, J. and Solomon ,M.M. (1999). *A Branch-First, Cut-Second Approach for Locomotive Assignment*. *Management Science*, 45, pp. 1156-1168