# Use of Integral Copulation to Fully Discrete the Field of Coordinates of Cartesius Urgent and in Polar Coordinates 

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#### Abstract

In discussing the area of the field, we will use two coordinate systems, namely the Cartesian Upright Coordinate (Coordinate Elbow) and the Polar Coordinate. Area of the Cartesian Upright Coordinate There are four kinds: (1). The area of the area passed by the curve $y=f(x)>0$, the $x$-axis, the line $x=a$ and the open $x=b$, (2). The area of the area passed by the curve $y=f(x)<0$, the $x$-axis, the line $x=a$ and the open $x=b$, (3). The width between the two curves $y_{1}=f_{1}(x)$ and $y_{2}=f_{2}(x)$; (4). The area of the field passed by the curve $x=f(y)>0$, the $y$-axis, line $y=$ $c$ and line $y=d$. depending on the area of the curve in the polar form $r=f(\theta)$ between two radians with $\theta=\theta_{1}$ and $\theta=\theta_{2}$. In this research we will discuss the use of integral for wide field at Upright Cartesius Coordinate and at Polar Coordinate.


Keywords: Upright Cartesian Coordinate, Polar Coordinate, Integral Count

## I. BACKGROUND

To find the area of the plane in vertical cartesius coordinates (rectangular coordinates) and at the polar coordinates an integral count is used, ie a typical integral or a particular integral. The existence of a particular Integral is that if f is continuous at $[\mathrm{a}, \mathrm{b}]$ then f can be integrated (integrable) at interval $[\mathrm{a}, \mathrm{b}]$. According to certain integral calculations:

$$
\int_{a}^{b} f(x) d x=-\int_{b}^{a} f(x) d x
$$

a. Research Question

How to settle area in Cartesian Upright coordinates and Polar Coordinates with the use of certain integral or integral counts.

## b. Purpose

The purpose of this problem is to find solutions on the use of integral counts to complete the area of the Upper Cartesian Coordinate and to the Polar Coordinate to facilitate the understanding of other mathematical subjects

## II. LITERATURE REVIEW

According Wardiman, when viewed from the verb that is integrate, it is known that the integral has two terms that is to say the number of or the whole of and get a function derivative has been known.
According to Wardiman, we simply call it the integral of $f(x)$ to $x$. The symbols are written $\int f(x) d x=F(x)+c$ with:

- written $\int$ is an integral notation
- $f(x)$ is integrally an integral function
- $d x$ is a differential integrator ie to what variable we will integrate
- $\quad \mathrm{F}(\mathrm{x})+\mathrm{c}$ is the result of the integration process with c integral constant
- Based on the definition of the above integral and the basic formula of derived functions, the following basic integral formulas are obtained:
- Integral Formulas

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1. \(\int a d x=a x+c ; a=\) constants
2. \(\int x^{n} d x=\frac{1}{n+1} x^{n+1}+c ; n \neq-1\)
3. \(\int e^{x} d x=e^{x}+c\)
4. \(\int a^{x} d x=\frac{a^{x}}{\ln a}+c ; a=\) positive
5. \(\int \frac{1}{x} d x=\ln |x|+c\)
6. \(\int \sin x d x=-\cos x+c\)
7. \(\int \cos x d x=\sin x+c\)
8. \(\int \sec ^{2} x d x=\tan x+c\)
9. \(\int \operatorname{cosec}^{2} x d x=-\cot x+c\)
10. \(\int \sec x \tan x d x=\sec x+c\)
11. \(\int \operatorname{cosec} x \cot x d x=-\operatorname{cosec} x+c\)
12. \(\int \tan x d x=\ln |\sec x|+c\)
\(=-\ln |\cos x|+c\)
13. \(\int \cot x d x=\ln |\sin x|+c\)
    \(=-\ln |\operatorname{cosec} x|+c\)
14. \(\int \sec x d x=\ln |\sec x+\tan x|+c\)
15. \(\int \operatorname{cosec} x d x=\ln |\operatorname{cosec} x-\cot x|+c\)
16. \(\int \frac{d x}{a^{2}+x^{2}}=\frac{1}{a} \arctan \frac{x}{a}+c\) when \(\mathrm{a}=\)
constants
17. \(\int \frac{d x}{\sqrt{a^{2}-x^{2}}}=\arcsin \frac{x}{a}+c\) when \(\mathrm{a}=\)
constants
18. \(\int \frac{d x}{x \sqrt{a^{2}-x^{2}}}=\frac{1}{a} \operatorname{arcsec}\left|\frac{x}{a}\right|+c\) when \(\mathrm{a}=\)
constants
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According to Sri Rejeki Dwi Putranti, if a any constants of $\mathrm{f}(\mathrm{x}), \mathrm{g}(\mathrm{x})$ is any function in x then:

1. $\int a f(x) d x=a \int f(x) d x$
2. $\int(f(x)+g(x)) d x=\int f(x) d x \pm \int g(x) d x$
 integrating techniques are required. According to Ayres Jr., Frank, will be discussed about the use of integral itself to be able to solve various mathematical problems that is about a certain integral. For example $f$ is a function identified at intervals $[\mathrm{a}, \mathrm{b}]$. However, we first define the norm of a partition $\mathrm{P}(\|\mathrm{P}\|)$, which is defined. As the longest subinterval of the P partition or can be expressed as $\mathrm{P}(\|\mathrm{P}\|)=\max \Delta \mathrm{xi}$.

According to Erwin Kreyszig, an integral is defined by using the Riemann summation. Let f be a function defined at intervals [a, b].. If $\lim _{\|P \rightarrow 0\|} \sum_{i=1}^{n} f\left(\varepsilon_{i}\right) \Delta x_{i}$ exists then f is called integrable at the interval [a, b]. Furthermore, if the limit
is $\lim _{\|P \rightarrow 0\|} \sum_{i=1}^{n} f\left(\varepsilon_{i}\right) \Delta x_{i}$ is called the definite integral of f at interval $[\mathrm{a}, \mathrm{b}]$. And usually expressed as $\int_{a}^{b} f(x) d x$ (reads integral f from a to b) or $\int_{a}^{b} f(x) d x=\lim _{\|P \rightarrow 0\|} \sum_{i=1}^{n} f\left(\varepsilon_{i}\right) \Delta x_{i}$

1. According to Seymour Lipschutz, the particular Integral Existing theorem is that if f is continuous at interval [ $\mathrm{a}, \mathrm{b}$ ] then f can be integrated (integrable) at interval $[\mathrm{a}, \mathrm{b}$ ]
2. The following are some special summations that may be useful for calculating the integral of course by using the definition:
3. $\sum_{i=1}^{n} i=1+2+3+\cdots+n=\frac{n(n+1)}{2}$
4. $\sum_{i=1}^{n} i^{2}=1^{2}+2^{2}+3^{2}+\cdots+n^{2}=\frac{n(n+1)(2 n+1)}{6}$
5. $\quad \sum_{i=1}^{n} i^{3}=1^{3}+2^{3}+3^{3}+\cdots+n^{3}=\left[\frac{n(n+1)}{2}\right]^{2}$
6. $\sum_{i=1}^{n} i^{4}=1^{4}+2^{4}+3^{4}+\cdots+n^{4}=\frac{n(n+1)\left(6 n^{3}+9 n^{2}+n-1\right)}{30}$

The properties are integral of course

1. $\int_{a}^{b} f(x) d x=0$
2. $\int_{a}^{b} f(x) d x=-\int_{b}^{a} f(x) d x$
3. $\int_{a}^{b} c f(x) d x=c \int_{a}^{b} f(x) d x$ with $\mathrm{c}=$ constants
4. $\int_{a}^{b}[f(x)+g(x)] d x=\int_{a}^{b} f(x) d x+\int_{a}^{b} g(x) d x$
5. $\int_{a}^{b}[f(x)-g(x)] d x=\int_{a}^{b} f(x) d x-\int_{a}^{b} g(x) d x$
6. $\int_{a}^{b} f(x) d x \geq 0$ if $\mathrm{f}(\mathrm{x}) \geq 0$ at $[\mathrm{a}, \mathrm{b}]$

According to Spiegel, Murray R, in the preceding material the assumptions for an integral sought function necessarily by the summation process of Riemann is a continuous function at intervals. The assumption states that functions that can be integral are necessarily continuous functions in infinite integration intervals. If we integrate by ignoring this assumption, then the integral is called an improper integral. There are two types of improper integrals:

1. Unnatural integral with infinite integration boundaries:
a. The upper limit of this integration is not up to

$$
\int_{a}^{\sim} f(x) d x=\lim _{b \rightarrow \sim}^{\sim} \int_{a}^{b} f(x) d x
$$

b. Uncompromising bottom limit

$$
\int_{\sim}^{b} f(x) d x=\lim _{a \rightarrow \sim} \int_{a}^{b} f(x) d x
$$

c. The lower limit and upper limit of unlimited integration

$$
\begin{aligned}
\int_{-\sim}^{\sim} f(x) d x & =\int_{-\sim}^{c} f(x) d x+\int_{c}^{\sim} f(x) d x \\
& =\lim _{a \rightarrow \sim}^{c} \int_{a}^{c} f(x) d x+{ }_{b \rightarrow \sim}^{\lim } \int_{c}^{b} f(x) d x
\end{aligned}
$$

1) An unnatural integral with an integral has a discontinuous point:
a. Integran is not continuous at the upper limit point of integration
b. Integran is not continuous at the lower limit point of integration
c. Integran is not continuous at the point $\mathrm{x}=\mathrm{c}$ in the integration interval $(\mathrm{a} \leq \mathrm{c} \leq \mathrm{b})$

$$
\begin{aligned}
\int_{a}^{b} f(x) d x & =\int_{a}^{c} f(x) d x+\int_{c}^{b} f(x) d x \\
& =\lim _{t \rightarrow c} \int_{a}^{t} f(x) d x+\lim _{s \rightarrow b} \int_{s}^{b} f(x) d x
\end{aligned}
$$

## III. METHOD

The method used is: Integration technique is a technique used in integrating functions performed by changing the form of integrants into forms contained in the integral basic formula. There are many integration techniques that can be used to solve an integral problem. In this material, several integration techniques will be discussed including substitution of algebraic and trigonometric properties and techniques commonly used in text books for an engineer.
1 Change to Basic Forms of Substitution
For example, given the integral integral problem problem $\int f(x) d x$, where the integral of the integrative function is not found in the integral basic formula, then in this substitution technique a transformation of the variable x into another variable (eg t) is carried out so that $\int f(x) d x=\int g(t) d x$, and $\int g(t) d t$ is found in the integral formula

In addition to transforming variables, we must also perform integrator differential transformations from dx to dt. To get the relationship dx and dt can be obtained by decreasing x against t .
2). Using Algebraic Operations and Trigonometry Identities. The algebraic operation referred to in this integration technique is to complete squares, division and multiplication of forms 1 . While the trigonometric identities are listed in the previous theory.
Completing squares is used to resolve integrals whose integrals contain the form of quadratic functions $a x^{2}+b x+c$ with $\mathrm{a}, \mathrm{b}$, and c being constants and no integral basic formulas. Thus, one technique that can be used is to change the shape of perfect squares.

$$
a x^{2}+b x+c=\left(x+\frac{b}{2 a}\right)+\left(\frac{b^{2}-4 a c}{4 a}\right)
$$

3). For Integrity of Trigonometry Functions

In this discussion the integration is in the form of trigonometric functions in the form of:
a. $\sin ^{m} x \cos ^{n} x$
b. $\tan ^{m} x \sec ^{n} x$
c. $\cot ^{m} x \operatorname{cosec}^{n} x$

For integran in the form $\sin ^{m} x \cos ^{n} x$ Substitution can be used as follows
$>$ If it is odd, then use substitution $\mathrm{t}=\cos \mathrm{x}$
$>$ If $n$ is gasal, then use substitution $t=\sin x$
$>$ If m is even and even, then trigonometric identities are used

$$
\begin{array}{r}
\sin ^{2} \mathrm{x}=\frac{1-\cos 2 x}{2} \\
\cos ^{2} x=\frac{1+\cos 2 x}{2}
\end{array}
$$

## 4). Trigonometry Substitution Technique

For example, the integral contains one of the irrational forms $\sqrt{a^{2}-x^{2}}, \sqrt{x^{2}-a^{2}}$ and $\sqrt{a^{2}+x^{2}}$ with a an arbitrary constant. In this technique trigonometric substitution is done so that the irrational form becomes a rational form. Trigonometric substitution for the above irrational forms is as follows:

| Form | Substitution |
| :--- | :--- |
| $\sqrt{a^{2}+x^{2}}$ | $\mathrm{X}=\mathrm{a} \sin \mathrm{t}$ |
| $\sqrt{x^{2}+a^{2}}$ | $\mathrm{X}=\mathrm{a} \sec \mathrm{t}$ |
| $\sqrt{a^{2}+x^{2}}$ | $\mathrm{X}=\mathrm{a} \tan \mathrm{t}$ |

5). Partial Integral Technique: If we are faced with an integral problem that can not be solved by the techniques we studied, we can use a partial integral technique. This technique is obtained by integrating the derived formulas of the two-function result.
For example $\mathrm{u}=\mathrm{u}(\mathrm{x})$ and $\mathrm{v}=\mathrm{v}(\mathrm{x})$ functions are differensiable in x
$\frac{d}{d x} \mathrm{uv}=\mathrm{u} \frac{d v}{d x}+\mathrm{v} \frac{d u}{d x}$ or $\mathrm{u} \frac{d v}{d x}=\frac{d}{d x} u v-v \frac{d y}{d x}$ and
$\int\left(u \frac{d v}{d x}\right) d x=\int\left(\frac{d}{d x}(u v)\right) \mathrm{dx}-\int\left(v \frac{d u}{d x}\right) \mathrm{dx}$
So the last equation can be written as follows:

$$
\int u d v=u v-\int v d u
$$

To use partial integral techniques it is necessary to note the following:

1. The use of this technique is to state integrants in two parts, one part as $u$ and the rest together with $d x$ as dv
2. When doing step 1 , select the dv section that can be integrated
3. It should be considered that $\int v d u$ is no more difficult than $\int u d v$

The integral on the right side is taken to the left segment so that:

$$
\begin{gathered}
\int e^{x} \sin x d x+\int e^{x} \sin x d x=-e^{x} \sin x+c \\
2 \int e^{x} \sin x d x=-e^{x} \cos x+e^{x} \sin x+\mathrm{c} \\
\int e^{x} \sin \mathrm{xdx}=\frac{1}{2}\left(-e^{x} \cos x+e^{x} \sin x\right)+\mathrm{c}
\end{gathered}
$$

## IV. ANALYSIS AND DISCUSSION

A. Integral Calculate Definition: Integral is another important branch of calculus which is the reverse operation of the derivative. If viewed from the verb which is integration, then it is known that integrals have two definitions:

1. Declare the amount of or all of and
2. Get a function whose derivatives are known.
B. Integral as a Meaning of Derivatives: For example, if given a function $f(x)$, then for a particular interest can be determined the derivative of the function $\mathrm{f}(\mathrm{x})$ is $\frac{d f(x)}{d x}$. On the contrary, it is not uncommon to find problems to find the origin function whose derivatives are known. For example: specify the function $f(x)$ which has an instance: $\mathrm{f}(\mathrm{x})=\frac{d}{d x} \mathrm{f}(\mathrm{x}) \cos \mathrm{x}$. The solution is $\mathrm{f}(\mathrm{x})=\sin \mathrm{x}$ is one answer to that question because $\frac{d(\sin x)}{d x}=\cos x$. In this case we call $f(x)$ the anti-derivative of $f(x)$. However, there are still many functions whose derivative is $\cos x$. the following:
$\Rightarrow \mathrm{f}(\mathrm{x})=\sin +1$
$\Rightarrow \mathrm{F}(\mathrm{x})=\sin \mathrm{x}-2$
$\Rightarrow \mathrm{F}(\mathrm{x})=\sin \mathrm{x}+1,2$
$\Rightarrow \mathrm{F}(\mathrm{x})=\sin \mathrm{x}+4 \pi$ and others
If these functions are also anti-derivatives of $\cos x$, the difference is the constant. Thus, the most appropriate answer to the question is $f(x)=\sin x+c$ with $c$ is an arbitrary constant. Henceforth, we simply call it the integral of $f(x)$ to $x$. In symbol, it is written $\int F(x) d x=F(x)+c$
B. Integral Count Usage: Multiple Areas with Integration. In discussing the area of a field, two types of coordinate system are used, ie Carttesius Upright coordinate (Coordinate Elbow) and Polar coordinates.
3. Area of Upright Cartesian Coordinate
a. The area of the field bounded by the curve $y=f(x)>0$, $x$ axis, line $x=a$ and line $x=b$ is:

a. Area of the bounded by the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})<0$, the x axis, line $\mathrm{x}=\mathrm{a}$ and line $\mathrm{x}=\mathrm{b}$ are:

c. The area of the bounded area by the curve $x=f(y) 0>0$, the $y$-axis, the line $Y=c$ and line $y=a$ are:

d. The area of the field bounded by the curve $x=f(y)<0$, $y$ axis, line $y=c$ and line $y=d$ are:

$e$. The area of the two curves $x_{1}=f_{1}(y)$ and $x_{2}=f_{2}(y)$ are:


$$
\mathrm{L}=\int_{c}^{d}\left(x_{1}-x_{2}\right) \mathrm{dy}
$$

f. Area of Field At Polar Coordinates

The area of the plane bounded by the curve in the polar form $r=f(\theta)$ between two radii with $\quad \theta=\theta_{1}$ and $\theta=$ $\theta_{2}$ is:


Example 1
Find the exact area of curve $y=x^{2}, x$ axis, line $x=1$ and line $x=3$
Solution :


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Example 2
Determine the area located above the $x$-axis and below the parabola $=4 x-x^{2}$
Solution


$$
\begin{aligned}
L & =\int_{0}^{4} y^{d x} \\
& =\int_{0}^{4}\left(4 x-x^{2}\right) d x \\
& =\left[2 x^{2}-\frac{1}{3} x^{3}\right]_{0}^{4} \\
& =32-\frac{64}{3} \\
& =10 \frac{2}{3} \text { unit area }
\end{aligned}
$$

Example 3
Determine the area of the field bounded by curves $y=x^{2}-4$ and $x$ axis
Solution


$$
\begin{gathered}
\mathrm{L}=-\int_{-2}^{2} y \mathrm{dx} \\
=-\int_{-2}^{2}\left(x^{2}-4\right) \mathrm{dx} \\
=-\left[\frac{1}{3} x^{3}-4 x\right]_{-2}^{2} \\
=-\left(\left(\frac{8}{3}-8\right)-\left(-\frac{8}{3}+8\right)\right) \\
=\frac{32}{3} \text { unit area }
\end{gathered}
$$

Example 4
Determine the area of the area bounded by curves $y=6 x-x^{2}$ and $y=x^{2}-2 x$
Solution


Example 5
Determine the area bounded by the curve $x=1+y^{2}$ and the line $x=10$
Solution


$$
\begin{aligned}
\mathrm{L} & =\int_{-3}^{3}\left(10-1-y^{2}\right) \mathrm{dy} \\
& =\int_{-3}^{3}\left(9-y^{2}\right) \mathrm{dy} \\
& =\left[9 y-\frac{1}{3} y^{3}\right] 3 \\
& =(27-9)-(-27+9) \\
& =36 \text { unit area }
\end{aligned}
$$

Example 6
Find the bounded area of the curve $\mathrm{y}=\sin \mathrm{x}, \mathrm{x}$ axis, from $\mathrm{x}=0$ to $\mathrm{x}=2 \pi$
Solution


$$
\begin{aligned}
& \mathrm{L}=\mathrm{L}_{\mathrm{I}}+\mathrm{L}_{\mathrm{II}} \\
& =\int_{0}^{\pi} \sin x d x+\left|\int_{\pi}^{2 \pi} \operatorname{sinx} d x\right| \\
& =[-\cos x]_{0}^{\pi}+[\cos x]_{\pi}^{2 \pi} \\
& =-(\cos \pi-\cos 0)+(\cos 2 \pi-\cos \pi) \\
& =4 \text { unit area }
\end{aligned}
$$

Example 7
Determine the area bounded by the ellipse with the parameter equation $x=a \cos t$ and

$$
y=b \sin t
$$



Example 8
Find the area of the cycloid curvature $x=\theta-\sin \theta$ and $y=1-\cos \theta$
Solution


$$
\begin{aligned}
\mathrm{X} & =\theta-\sin \theta \\
\mathrm{dx} & =(1-\cos \theta) d \theta \\
\mathrm{~L} & =\int_{0}^{2 \pi} y d x \\
& =\int_{0}^{2 \pi}(1-\cos \theta)(1-\cos \theta) d \theta \\
& =\int_{0}^{2 \pi}\left(\frac{3}{2}-2 \cos \theta+\frac{1}{2} \cos 2 \theta\right) d \theta \\
& =\left[\frac{3}{2} \theta-2 \sin \theta+\frac{1}{4} \sin 2 \theta\right]_{0}^{2 \pi} \\
& =\left(\frac{3}{2} .2 \pi-2 \sin 2 \pi+\frac{1}{4} \sin 4 \pi\right)-0 \\
& =3 \pi \text { unit area }
\end{aligned}
$$

Example 9
Determine he bounded area of cardioida $r=1+\cos \theta$
Solution


$$
\begin{aligned}
\mathrm{L} & =\quad \int_{\theta_{1}}^{\theta_{2}} r^{2} d \theta \\
& =\quad \int_{0}^{\pi}(1+\cos \theta)^{2} \mathrm{~d} \theta \\
& =\quad \int_{0}^{\pi}\left(1+2 \cos \theta+\cos ^{2} \theta\right) d \theta \\
& =\left[\theta+2 \sin \theta+\frac{1}{4} \sin 2 \theta\right]_{0}^{\pi} \\
& =\left(\pi+2 \sin \pi+\frac{1}{4} \sin 2 \pi\right)=0 \\
& =\pi \text { unit area }
\end{aligned}
$$

Example 10
Find the curve curve area $y^{2}=x^{4}(4+x)$
Solution


$$
\begin{aligned}
& \mathrm{L}= 2 \int_{-4}^{0} y d x \\
&=2 \int_{-4}^{0} x^{2} \sqrt{4+x} \mathrm{dx} \\
& \text { Misal }: z^{2}+4+\mathrm{x} \rightarrow x=z^{2}-4 \\
& \quad \mathrm{dx}=2 \mathrm{z} \mathrm{dz} \\
& \mathrm{~L}= 2 \int_{-4}^{0} x^{2} \sqrt{4+x} d x=2 \int_{-4}^{2}\left(z^{2}-4\right)^{2} \cdot z \cdot 2 z d z \\
&= 4 \int_{-4}^{0}\left(z^{6}-8 z^{4}+16 z^{8} \mathrm{dz}=4\left[\frac{2^{7}}{7}-\frac{8 z^{5}}{5}+\frac{6 z^{3}}{3}\right]_{-4}^{0}\right. \\
&= \frac{4096}{1056} \text { unit area }
\end{aligned}
$$

## V. CONCLUSION

To be able to complete the use of integral counts for the area of the Cartecius Upright coordinate and the polar coordinates need to be considered:
1). Draw a curve to get the integral boundaries.
2). Size of field (L) sought, preferably shaded.
3). Can use the integral formulas of course, to find the area of the field (L).

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