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Relation Between Limit of Function and Derivative

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Abstract
This section will introduce the concept of Limit intuitively by illustrating some problems. From that, we are able to define the Limit function of a value then the relation between Limit and derivative can be discussed. At first, we should know about the rate of change which can happen to several occurrences how to calculate the Limit with L'Hospital Theory (by using derivative).
Keywords: *Limit of Function, Derivative*

I . INTRODUCTION

A. Background

The concept of Limit is the basic concept and very important in mathematic, mainly in calculus. Differential and integral are part of Limit. It means, if we could understand Limit, we would solve the other material of calculus easily. This selection also discuss the Limit and derivative theory, because they are really needed to solve some cases. To get the solutions, we need mathematical theory and formula which are related to Limit of function and derivative.

B. Research Question

How is the relation between Limit of function and derivative by counting the Limit with L'Hospital theory?

C. Purpose

The purpose of this section is to acquire the solutions about the relation between Limit and derivative.

II. LITERATURE REVIEW

According to Frank Ayres, Jr., definition of Limit which is obtained from an observation is; $\lim_{x\to a} f(x) = L$, It implies, if x approaches a, but is not equal to a , then $f(x)$ can be made to be as close to L.

Afterwards, the definition of derivative is a function which is a rate of change from something towards something and a part of calculus. It has been used a lot for calculating speed, pace, and etc

As Murray R. Spiegel had written that the solution of Limit of indefinite from was obviously done in two phases. First, Limit of indefinite from becomes Limit of definite form and the second was to make a substitution of x. The solution of Limit oftenly uses L'Hospital theory in derivative.

Before we discuss the derivative more, we will be introduced the velocity of an object in several cases which can be measured by dividing the range and the time required in km/hour. Derivative is still needed to solve some cases. For example, the Chief of a company wants to know how to combinate his products in purpose to gain the highest profits. To find the solution, the Chief needs mathematical theories and formulas that related to derivative. They are going to be discussed in this section.

The topics discussed regarding the derivative is how to determine the tangent gradient of a function and how to calculate the Limit with L'Hospital theory.

Murray R. Spiegel also wrote about the two types Limits that involving infinitive value which can happen to Limit or the value of x. They are infinity Limit and uninfinity Limit.

According to Frank Ayres, Jr., Limit theory is; $\lim_{x \to a} f(x) = L$ if $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$

Fortunately, those theories are useful for examining if the value of Limit of function exists or doesn't. To show that there is no limit of function at $x = a$, then how to do is using; If $\lim_{x \to a} f(x) \neq \lim_{x \to a} f'(x)$ then $\lim_{x \to a} f(x)$ doesn't exist.

III. METHOD

The following below is the definition of Limit which is obtained from an observation;

$lim\,(x) = L$, implies if (x) approaches a, but not equal to a, then f(x) $x\to a$ can be made to be as close to L.
The settlement of Limit indefinite form generally can be done in four stages;

1. Change the Limit indefinite form to Limit definite form.
   The changing of Limit indefinite form is done by:
   a. Multiply with form 1.
   b. Eliminate the causative factor of indefinite form by factoring.
2. Subtitute the value of (x) in Limit.
   a. Limit of indefinite form $\frac{0}{0}$ which is $\lim_{x\to 0}\frac{f(x)}{g(x)}$ with $\lim_{x\to 0} f(x) = 0$,
      and $\lim_{x\to 0} g(x) = 0$,
   b. Limit of indefinite form , $\frac{\pm\infty}{\pm\infty}$ .which is; $\lim_{x\to 0}\frac{f(x)}{g(x)}$ with $\lim_{x\to 0} f(x) = \pm\infty$ and $\lim_{x\to 0} g(x) = \pm\infty$
   c. Limit of indefinite value 0, $\pm\infty$ which is $\lim_{x\to 0}\frac{f(x)}{g(x)}$ with $\lim_{x\to 0} f(x) = 0$,
      changed and $\lim_{x\to 0} g(x) = \pm\infty$ To solve the limit of indefinite form $0, \pm\infty$ by firstly changing the form $f(x)g(x)\ \ to\ \ f(x)\frac{1}{1/g(x)}\ \ or\ f(x)\frac{1}{1/f(x)}$
   d. Limit of indefinite value $+\infty - \infty$ or $-\infty + \infty$ which is $\lim_{x\to 0} f(x)\ g(x)$, with $\lim_{x\to 0} g(x) = \pm\infty$
      To solve the limit of indefinite form $+\infty - \infty$ or $-\infty + \infty$ by firstly multiplying the form 1. It will change the limit of indefinite form $+\infty - \infty$ or $-\infty + \infty$ to indefinite limit.
   e. Limit of indefinite form 0°, $\pm\infty°$ and $1^{0\pm\infty}$ whose $\lim_{x\to 0} f(x)^{g(x)}$
      change the limit of indefinite form to limit of definite form 0, ±∞ by doing logarithm of the equation $y = f(x)^{g(x)}$ later on, make the limit of both segment. This solution usually uses L'Hospital theory.
3. L'Hospital Theory
   e.g. $\lim_{x\to a}\frac{f(x)}{g(x)}$. is indefinite form $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$ and, for example, f and g are able to use derivative which is containing a, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$
   If the limit exists or equals to $\pm\infty$.
4. Limit involving infinity value
   There are 2 types of limits involving the infinity value:
   1. Infinity limit is limit of function *f*,*x*. whose values is heading to $+\infty$ or $-\infty$ when x towards a or $\lim_{x\to a} f(x) = \pm\infty$
      x can also toward $+\infty\ or\ -\infty$ and the value of $+\infty$ is written as ∞.
   2. Infinity limit is limit of function $f(x)$ for *x* going to $+\infty$ or $-\infty$ written with $\lim_{x\to a} f(x) = 1$ or $\lim_{x\to a} f(x) = L$

IV . ANALISIS AND DISCUSSION

A. Definition of Limit

The following below is an illustrastion of problems to introduce the definition of limit.

Take a look at the function ; $f(x) = \frac{2x^2-5x-3}{x-3}$, $x \neq 3$ How much is the value of function *f* when *x* close to 3?
Completion;
When *x=3*, the denominator is 0, so that *f*,*3*. is undefined. But for *x≠3* the value of *f*,*x*. exists. What happens to the value of the function *f* around *x=3*? By calculating numerically the value of *f* function around *x=3* in obtaining the table as follows.

Table 1. The value of *f* function around *x=3*

x	2.9	2.99	2.999	...	...	3	...	...	3.001	3.01	3.1
f,*x*.	6.9	6.98	6.998	...	...	7	...	...	7.002	7.02	7.2

The first row of the table is given a value of *x* close to 3 from the left and from the right. The second row of the table shows the value of the *f* function that is obtained by entering the value of *x* from the first row and approaches 7.

This condition can be written as: $\lim_{x\to 3} f(x) = 7\ \ or\ \lim_{x\to 3} f(x)\frac{2x^2-5x-3}{x-3} = 7$

B. The Completion Limit of Indefinite Form

The calculation of the value of function $f(x) = \frac{2x^2-5x-3}{x-3},\ x \neq 3$ from the example above is done for *x⟶3.* The calculation becomes a problem when *x⟶3,* the value of the denominator and the numerator of the function will be 0 or the value of the current function change to $\frac{0}{0}$ (Limit of indefinite form).

C. Definition of Derivative

Derivative is a function which is a rate of change from something towards something and a part of calculus. It has been used a lot for calculating speed, pace, and etc. For example, first, on the situation of an object moving along a path which can be measured its average speed is the distance of an object moving divided by the time required to get through that distance (km/hour). Second, in the breeding of bacteria, we can measure the growth rate of the population (percent/day). Third, the rainfall gauge is able to measure the rainfall every month (cm/month). Generally, the avarage rate of change in a certain period is the total of change divided by the time that needed for changing.

The example; a car moves through certain road and takes t second after the first move is $s(t)$ It takes 400 meters in 40 seconds and takes 800 meters in 80 seconds. Then, the car's velocity that has been taken from the 40th to the 8th seconds is:

$AvarageVelocity:$

$$\frac{\Delta s}{\Delta t} = \frac{800-400}{80-40} = 10m/sec$$

Now, the question is, how many velocity that has been traveled for 40 seconds? The instantaneous velocity of the car at $t = 40$ is expressed as:

$\lim\limits_{t \to 40} \frac{s(t)-s(40)}{t-40}$

Generally, the instantaneous velocity at $t = 40$ is expressed as:

$\lim\limits_{t \to a} \frac{(s)t-s(a)}{t-a}$..(1)

Or, if $t = a + b$, then the velocity that has been traveled when $t = a$ is:

$\lim\limits_{t \to a} \frac{s(a+b)-s(a)}{b}$ ....(2)

The process above is called differential and the result is called derivative. Mathematicaly, derivativ is defined as:

- Derivativ of f function that x is inside the domain f:
  $f(x) = \lim\limits_{\Delta x \to 0} \frac{f(x)+\Delta x-f(x)}{\Delta x}$ (If the Limit exists.).......................(3)

If the limit in equation (3) exists, then f derivates to x, the derivative of a function on certain point is defined as:

- If x is a number inside the domain f, then the derivative of f to $x = x_t$ is:
  $f(x_t) = \lim\limits_{\Delta x \to 0} \frac{f(x_t+\Delta x)-f(x_t)}{\Delta x}$ (If the Limit exists.)............................(4)

  Or, if $\Delta x = x_t$ then:
  $f(x_t) = \lim\limits_{\Delta x \to 0} \frac{f(x)-f(x_t)}{x-x_t}$ (If the Limit exists.).................................(5)

Notation that oftenly used for derivative function $y = f(x)$ is:

1. $f'(x)$ or y' : Lagrange Notation
2. $\frac{df}{dx}$ or $\frac{dy}{dx}$: Leibniz Notation
3. $Dx(f)$: Operator D Notation
4. Calculating

Example 1: Determine the result of

$\lim\limits_{x \to 1} \frac{x-1}{x^2-1} x \to 1$

Solution:

- If it is done by substitution and becomes undefined:

$$\lim_{x\to 1}\frac{x-1}{x^2-1} = \frac{1-1}{1^2-1} = \frac{0}{0}$$

Therefore, $\lim_{x\to 1}\frac{x-1}{x^2-1}$ can be solved by using L'Hospital theory, so then;

$$\lim_{x\to 1}\frac{x-1}{x^2-1} = \lim_{x\to 1}\frac{1}{2x} = \frac{1}{2}$$

Example 2: Determine the result of:

$$\lim_{x\to 0}(\frac{1}{\sin x} - \frac{1}{x})$$

Solution:

- If it is done by substitution and becomes undefined:

$$\lim_{x\to 1}\left(\frac{1}{\sin x} - \frac{1}{x}\right) = \frac{1}{\sin o} - \frac{1}{0} = \frac{1}{0} - \frac{1}{0} = \infty - \infty$$

Therefore, to calculate the limit above is by making a change from

$\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right)$ to $\lim_{x\to 0}\frac{x-sin\ x}{x\sin x} Lim\frac{x-\sin x}{x\ sin\ x}$ (If by doing the substitution obtained $\frac{0}{0}$)

So then, it can be done by using L'Hospital theory.

$$\lim_{x\to 0}\left(\frac{1}{\sin x} - \frac{1}{x}\right) = \lim_{x\to 0}\frac{x-sinx}{x\sin x} Lim\ \frac{x-\sin x}{xsin\ x}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{\sin x + x\cos x}$$

$$= \lim_{x\to 0}\frac{\sin x}{2\cos x + x\sin x}$$

$$= \frac{0}{2cos0 - 0sin0}$$

$$= \frac{0}{2(1)-0(0)}$$

$$= \frac{0}{2}$$

$$= 0$$

Example 3 : Determine the result of

$$\lim_{x\to\infty} x^{\frac{1}{x}}$$

Solution:

- If it is done by substitution and becomes undefined:

$$\lim_{x\to\infty} x^{\frac{1}{x}} = \infty^{\frac{1}{\infty}} = \infty^{\circ}$$

Therefore, to calculate the limit above is by making a change from $y = \lim_{x\to\infty} x^{\frac{1}{x}}$ to:

$$y = \lim_{x\to\infty} x^{\frac{1}{x}}$$

$$= \lim_{x\to\infty} e^{x^{\frac{1}{x}}} Lim\ e^{x^{\frac{1}{x}}}$$

$$= e^{\underset{x\to\infty}{Lim}\, e^{x^{\frac{1}{x}}}}$$

Where $\lim_{x\to\infty} e^{x^{\frac{1}{x}}} = \lim_{x\to\infty}\frac{1}{x}\ln x = \lim_{x\to\infty}\frac{\ln x}{x}$

(If by doing the substitution obtained $\frac{0}{0}$)

So then, it can be done by using L'Hospital theory.

$$lim\ \frac{\ln x}{x} = lim\ \frac{\frac{1}{x}}{1}$$

$$= \frac{0}{1}$$

$$= 0$$

Therefore; $y = e^{x^{\frac{1}{x}}} = e^{\circ} = 1$

V. CONCLUSION

1. If the limit is done by dirrect substituion until undefined ($\frac{0}{0}$), it can be solved with L'Hospital Theory.
2. If the limit is done by dirrect substitution until undefined, such as $\infty-\infty$, $\infty+\infty$, and $\pm\infty^{\circ}$, it must be changed first. Then, if it becomes $\frac{0}{0}$, it also can be solved with L'Hospital Theory.
3. To solve the limit of inderterminate form such as o° *and* $1^{\pm\infty}$, can be done with these several phases:
   - From the principal exponential $f(x)^{g(x)} = e^{ln\, f(x) g(x)}$
   - Limit both segments, then obtained:

$$\lim_{x\to 0} f(x)^{g(x)} = \lim_{x\to 0} e^{ln\, f(x) g(x)}$$

From this segment can be obtained $\lim_{x\to 0} f(x)^{g(x)} = \lim_{x\to 0} e^{ln\, f(x) g(x)}$

- From natural logarithm can be obtained

$$e^{\lim_{x\to a} f(x)^{g(x)}} = e^{\lim_{e\to a} e^{lnf(x)g(x)}}$$

- The solution of $lim\ g(x)\ m\ f(x)$ is using L'Hospital Theory.

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