Settlement of Differential Equality with Laplace Transformation

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Abstract
The Laplace transform of the simplest function F (t) is the function f (S) which can be expressed in the form : f(s) = $\int_0^\infty e^{-st}$ *.....(1) if this interval exists. Here a is taken real. The form (1) is often also written in form : L [f(t)] = f(s).....(2) according to the usual differential count :* $\int_0^\infty e^{-st}$ *f(t) dt =* $\lim_{u\to\infty}\int_0^\infty e^{-st}$ *f(t) dt.....(3) as a function f (t) is called the exponential order function on* $[o,\infty]$*, if there are constants M and a, M > 0 so* $|f(t)|<e^{at}$ *for everyone t > 0. In this research will be discussed the solution of differential equation with Laplace transform, through approach of linear equation of n-level differential with constant coefficient*
Keywords: *Differential equations, Laplace transformation*

I. INTRODUCTION

A. Background

The Laplace transform of the function f (t) is the function f (s), expressed by the form:

F(s)= $\int_0^{\sim} e^{-st}$ F(t) dt ….. (1)

If this integral exists. In this case a is taken in real terms. the form (i) is often written in the form of (i) often written in form

L [f(t)] = f(s) ….. (2)

According to the usual differential calculation:

$\int_0^{\sim} e^{-st}$ F(t) dt = $\lim_{\mu\to\sim}\int_0^{\sim} e^{-st}$ F(t) dt ….. (3)

B. Research Question

How to solve differential equations with Laplace transform by finding conditional settlement of the linear equation of n-level differential with constant coefficient?

C. Purpose

The purpose of this problem is to find a solution about solving differential equations with Laplace transforms to make it easier to understand other mathematical materials.

II. LITERATURE REVIEW

According to Ayres JR, Frank, given the function f (t) defined for all t ≥ 0, multiplied by f(t) e^{-st} and integral to t from zero to threshold, then when the integral exists, from SF (s) = f (t) dt. The function F (s) with these variabels is called the Laplace transform of the function f (t).

According to Seymour Lipschutz, Laplace transform can be used to find the conditional settlement of the linear equation of the n-level differential equations with a constant coefficient

$$a_1\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \ldots\ldots\ldots\ldots\ldots + a_{n-1}\frac{dy}{dx} + a_ny = q(x) \;\ldots..\; (1)$$

provided that:

$$y(0) = b_o \;;\; y'(0) = b_1 \;;\; y''(0) = b_2 \;;\; \ldots\ldots\ldots\ldots\ldots\; y^{(n-1)}(0) = b_{n-1} \;\ldots..\; (2)$$

the formula used is : L $\left[\frac{d^ny}{dx^n}\right]$.

If L [y(x)] = Y (s) is assumed with continuous derivatives for x> 0, then:

$$L\left[\frac{d^n y}{dx^n}\right] = s^n y(s) - b_0 S^{n-1} - b_1 S^{n-2} - \ldots\ldots\ldots\ldots - b_{n-2} S - b_{n-1}$$

According to kreyzig Erwin, a function F (t) is called an exponential order function on [0,∼], if there are constants M and a, m> 0 so: $|F(t)| < M e^{at}$ for everyone t>0

According to Spiegel, Murray R, a function is called continuous part by part (Sectionally continuous or piecewise continuous) in intervals $a \le t \le b$, if this interval can be divided into finite-numbered part intervals and within each interval the function is continuous and has left and right limits

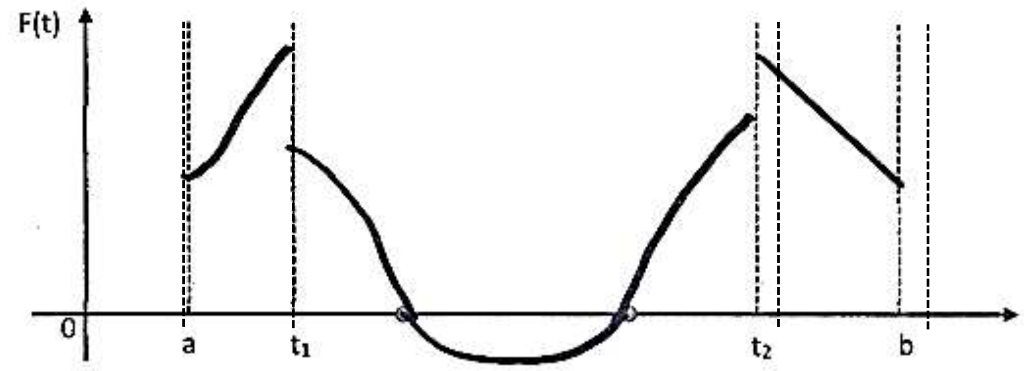

Figure 1. Sectionaly continuous orpiecewise continuous

According to Wardiman, the linearity of the Laplace transform (Linearity property), if c1, and c2 is constant, while F1 (t) and F2 (t) functions of the Laplace transformations are respectively f1 (s) and f2 (s), then :

$$L[c_1F_1(t) \pm c_2F_2(t)] = c_1L[F_1(t)] \pm c_2 L[F_2(t)] = c_1F_1(s) \pm c_2F_2(s)$$

According to Ayres JR, Frank, Laplace transforms from derivate ie if L[f(t)] = f(s), then :

$$L[F'(t)] = sf(s) - F(0)$$

Table 1. Laplace Transform Formulas

No	F(t)	L [F(t)] = f (s)	
1	1	$\frac{1}{s}$	s > 0
2	t	$\frac{1}{s^2}$	s > 0
3	t^2	$\frac{2!}{s^3}$	s > 0
4	t^n (n=1,2,……)	$\frac{n!}{s^{n+1}}$	s>0
5	e^{kt}	$\frac{1}{s-k}$	s > k
6	sin kt	$\frac{k}{s^2+k^2}$	s > 0
7	cos kt	$\frac{s}{s^2+k^2}$	s > 0
8	sinh kt	$\frac{k}{s^2-k^2}$	s>|k|
9	cosh kt	$\frac{s}{s^2-k^2}$	s>|k|

According to Wardiaman, when Laplace transforms a function F (t) is f (s), ie L [F(t)] = f (s), then F (t) is called the inverse Laplace transform of f (s) and is written :

L [F(t)] = L $^{-1}$ [f (s)]

Table 2. Inverse Laplace Transform

No	**F(s)**	F(t) = L $^{-1}$ [f (s)]
1	$\frac{1}{s}$	1
2	$\frac{1}{s^2}$	t
3	$\frac{2!}{s^3}$	t^2
4	$\frac{n!}{s^{n+1}}$	t^n
5	$\frac{1}{s-k}$	e^{kt}
6	$\frac{k}{s^2+k^2}$	sin kt
7	$\frac{s}{s^2+k^2}$	cos kt
8	$\frac{k}{s^2-k^2}$	sinh kt
9	$\frac{s}{s^2-k^2}$	cosh kt

III. METHODS

The method used is:

1. Laplace transform of some simple functions
   a. L[1] $= \int_0^{\sim} e^{-st}.1 \text{ dt} = \left[-\frac{1}{s}e^{-st}\right]_0^{\sim} = \frac{1}{s}$
   then : L[1] $=\frac{1}{s}$ for s > 0

   b. L[t] $= \int_0^{\sim} te^{-st} \text{ dt} = \left[-e^{-st}(\frac{1}{s}t + \frac{1}{s^2})\right]_0^{\sim} = \frac{1}{s^2}$
   then : L[1] $=\frac{1}{s^2}$ for s > 0

   c. L[t^2] $= \int_0^{\sim} t^2e^{-st} \text{ dt} = \frac{2!}{s^3}$
   then : L[t^2] $=\frac{2!}{s^3}$ for s > 0

   d. L[t^n] $= \int_0^{\sim} t^ne^{-st} \text{ dt}$

   Substitution: st = x
   dt = $\frac{1}{s}$ dx
   Boundaries:
   for t = 0 => x = 0
   T = ~ => x = ~
   Integral becomes=
   $\frac{1}{s^{n+1}} \int_0^{\sim} x^ne^{-st}\, dt = \frac{n!}{s^{n+1}}$
   then : L[t^n] = $\frac{n!}{s^{n+1}}$ for s > 0 and n > -1

   e. L[e^{kt}] $= \int_0^{\sim} e^{-st}e^{kt} \text{ dt}$
   $= \int_0^{\sim} e^{-(s-k)t} \text{ dt}$
   $= \left[-\frac{1}{s-k}\ e^{-(s-k)t}\right]_0^{\sim}$
   $= \frac{1}{s-k}$

then : L $[e^{kt}] = \frac{1}{s-k}$ for s > k

f. L[sin kt] $= \int_0^{\sim} e^{-st} \sin kt$ dt

$= \left[- \frac{-e^{-st} \sin kt}{s} - \frac{ke^{-st} \cos kt}{e^2}\right]_0^{\sim}$

$- \frac{k^2}{s^2} L\,[\sin kt]$

$= \frac{k^2}{s^2} - \frac{k^2}{s^2} L\,[\sin kt]$

then : $L\,[\sin kt] = \frac{k^2}{s^2+k^2}$ for s > 0

g. L[cos kt] $= \int_0^{\sim} e^{-st} \cos kt$ dt

$= \left[- \frac{-e^{-st} \cos kt}{s} - \frac{ke^{-st} \sin kt}{s^2}\right]_0^{\sim}$

$- \frac{k^2}{s^2} \int_0^{\sim} e^{-st} \cos kt$ dt

$= \frac{1}{s^2} - \frac{k^2}{s^2} L\,[\cos kt]$

then : $L\,[\cos kt] = \frac{s}{s^2+k^2}$ for s > 0

2. Used from Laplace transformations

a. Libearity Property
if c_1 dan c_2 constant, while $F_1(t)$ dan $F_2(t)$ functions that have their respective Laplace transforms $f_1(s)$ and $f_2(s)$, then :
$L[c_1f_1(t) \pm c_2f_2(t)] = c_1L[f_1(t)] \pm c_2L[f_2(t)]$
$= c_1f_1(s) \pm c_2f_2(s)$

b. Translation or shifting Property
(1) if L [F(t)] = f (s) then L [e^{at} F(t)] = f(s-a)

(2) if L [F(t)] = f (s) and G (t) = $\begin{cases} F(t-a); t>a \\ 0 \quad ; t<a \end{cases}$
then : L [G(t)] = e^{as} F(s)

c. Chance of scale property
if L [F(t)] = f (s) Then L [F(at)] = $\frac{1}{a} f(\frac{s}{a})$

d. Laplace transform from derivative
if L [F(t)] = f(s) then L [F'(at)] = s f(s) – F(0)

e. Laplace Transformasi From Integral
if L [F(t)] = f(s) hen L [$\int_0^t f(u)du = \frac{f(s)}{s}$

IV. ANALYSIS AND DISCUSSION

A. Definition of Differential Equations

Many important problems in engineering, physics and social sciences, when in mathematical form, require the determination of a function that satisfies a problem containing one or more derivatives of an unknown function. Such an equation is called Differential Equation. One of the differential equations we know is Newton's Law

$$M \frac{d^2u(t)}{dt^2} = F\,[t, u(t), \frac{du(t)}{dt}]$$

For the position of a particle u (t) subjected to force F, which may be a function of time t, position u (t) m and velocity $\frac{du(t)}{dt}$ to determine the movement of the particles subjected to the F force, a function is required that satisfies the equation.

B. Definition of Laplace Transformation

The Laplace transform of the function F (t) is a function f (s) expressed in the form: F(s) $= \int_0^{\sim} e^{-st}$ F(t) dt, if this integral exists and s real, then Laplace transform can be written in the form L[F9T0} = f(s), according to the usual differential count can be written = $\int_0^{\sim} e^{-st}$ F(t) dt = $\lim_{u \to \sim} \int_0^{u} e^{-st}$ F(t)dt Solving differential equations with laplace transforms.

Resolution of differential equations with Laplace transform

(1) The linear Differential Equation level n with constant coefficients

Laplace transformation can be used to observe the conditional settlement of the linear differential equation with the constant coefficient

$Q_0 \frac{d^n y}{dx^n} + Q_1 \frac{d^{n-1} y}{dx^{n-1}} + ... + Q_{n-1} \frac{dy}{dx} + a_n y = q(x)$ ….. (1)

With the provision of

$Y(0) = b_0$; $y'(0) = b_1$; $y''(0) = b2$; .... ; $y^{n-1}(0) = b_{n-1}$

By using the formula : $L\left[\frac{d^n y}{dx^n}\right]$ (2)

If presupposed L [y(x)] = y (s) with a continuous derivative derivative for x>o then :

$L\left[\frac{d^n y}{dx^n}\right] = s^n y(s) - s^{n-1} y(0) - s^{n-2} y'(0)................$

$= s\, y^{(n-2)}(0) - y^{(n-1)}(0)$

We enter the terms above, then there

$L\left[\frac{d^n y}{dx^n}\right] = s^n y(s) - b_0 S^{n-1} - b_1 s^{n-2} - .... - b_{n-2} S - b_{n-1}$

(2) How to solve Differential Equations

To find a conditional solution of a differential equation (1) with condition (2), first of all, is taken; the second laplace transform of the alloy of equation (1) then find an algebraic equation of y (s) solve this equation in y (s), then collected ivers Laplace transform to determine y (x)

$= L^{-1}[y(s)]$

Example 1

Complete the differential equation :

Y" + y = x, with the provision of y(0)=1 and y'(0)=-2

Solution

a. Using Laplace transform

Suppose L[Y(s)] = y(s) then :

L[y"+y] = L [x] or L y" + Ly = Lx

$S^2 y(s) - sy(0) - y'(0) + y(s) \quad = \frac{1}{s^2} (s^2+1)\, y(3)$

$= \frac{1}{s^2} + s - 2$

$y(s) = \frac{1}{s^2(s^2+1)} + \frac{s-2}{s^2+1}$

then the solution of the differential equation is:

$y(x) = L^{-1}[y(s)] \quad = L^{-1}[\frac{1}{s^2(s^2+1)}] + L^{-1}[\frac{s-2}{s^2+1}]$

$= L^{-1}[\frac{1}{s^2} - \frac{1}{s^2+1}] + L^{-1}[\frac{s}{s^2+1} - \frac{2}{s^2+1}]$

So the Solution is:

Y= x – 3 sin x + cos x

b. In the usual way

Differential equations: y" + y = x can be written (D^2 + 1) y = x

If $\frac{d}{dx}$ D (operator D), then the solution of the differential equation is

:

$y = c_1 \sin x + c_2 \cos x + \frac{1}{D^2+1} x$
$= c_1 \sin x + c_2 \cos x + x$
$y' = c_1 \cos x + c_2 \sin x + 1$

Example 2

Complete the differential equation
$y'' - y' - 2y = 4x^2$ with the provision of y(0) = 1 and y'(0) = 4

solution

by way of laplace transform:
$L[y''] - l[y'] - 2 L[y] = 4 L [x^2]$
$[s^2y(s) - sy(0) - y'(0) - [sy(s) - y(0) - 2y(s)] = \frac{8}{s^3}$
$(s^2 - s - 2) y(s) = \frac{8}{s^3}+s = 3$
So $y(s) = \frac{8}{s^3(s^2-s-3)} + \frac{s+3}{(s^2-s-3)}$

Then the inverse of Laplace transformation takes place, then there is:
$y = L^{-1} [y(s)]$
$= L^{-1}\left[\frac{8}{s^3(s^2-s-3)}\right] + L^{-1}\left[\frac{s+3}{s^2-s-3}\right]$
$= L^{-1}\left[\frac{8}{s^3(s+1)(s-2)}\right] + L^{-1}\left[\frac{s+3}{(s+1)(s-3)}\right]$
$= \left[\frac{8}{s^3(s-2)}\right]_{s=-1}^{e^{-x}} + \left[\frac{8}{s^2(s+1)}\right]_{s=2}^{e^{2x}} + \left[\frac{1}{2!}\frac{d^2}{ds^2}\left(\frac{8}{s^2--s-2}\right) + \frac{d}{ds}\left(\frac{8}{s^2-s-2}\right)x + \frac{8}{s^2-s-2}.\frac{x^2}{21}\right]_{s=0} + \left[\frac{s+3}{s-2}\right]_{s=-1}^{e^{-x}} + \left[\frac{s+3}{s+1}\right]_{s=2}^{e^{2x}}$

So the solution of the differential equation is:
$Y = (3 + 2x - 2x^2 + \frac{1}{3}e^{2x} + \frac{8}{3}e^{-x})+(-\frac{2}{3}e^{-x} + \frac{5}{3}e^{2x})$
$= 2e^{2x}+2e^{-x} - 2x^2 +2x - 3$

Example 3

determine the solution of the differential equation
$y'' - 3y' + 2y = 4e^{2x}$ with the provision of y(0)=3 and y'(0)=5

Solution

Taken laplace transform, then there are:
$L[y''] - 3L[y'] + 2L[y]=4L[e^{2x}]$
$[s^2 y(s) - s y(0) - y'(0)] - 3[s y(s) - y(0)] + 2y(s) \frac{4}{s-2}$
$(s^2 - 3 s + 2) y(s) = \frac{4}{s-2} - 35 + 14$

So

$Y(s) = \frac{4}{(s^2 -3s-2)(s-2)} - \frac{3s-14}{s^2 3 s-2}$
$= \frac{3s-14}{(s-1)(s-2)^2} - \frac{3s-14}{(s-1)(s-2)}$

Then taken inverse laplace transform, there are:
$Y = L^{-1} [y(s)]$
$= L^{-1}\left[\frac{y}{y(s-1)(s-2)^2}\right] - L\text{-}1\left[\frac{3s-14}{(s-1)(s-2)}\right]$
$= \left[\frac{4}{(s-2)^2}\right]_{s=1}^{e^x} + \left[\frac{d}{ds}\frac{4}{(s-1)} + \frac{4}{s-1}x\right]_{s=2}^{e^x} - \left[\frac{3s-14}{s-2}\right]_{s=1}^{e^x} - \left[\frac{3s-14}{s-1}\right]_{s=2}^{e^x}$

So the solution of the differential equation is:
$Y = 4e^x - 4e^{2x} + 4xe2^x + 8e^{2x} - 11e^2$
$Y = -7e^x + 4xe^{2x} - 4e^{2x}$

Example 4

Complete the differential equation:

$\frac{d^3y}{dx^3} - y = e^x$ provided, for x = 0 There y = 0, y' = 0 and y" = 0

Solution

When a laplace transform is taken for this equation, it is obtained

$L\left[\frac{d^3y}{dx^3}\right] - L[y] = L[e^x]$

$S^3 y(s) - s^2 y(0) - sy'(0) - y''(0) - y(s) = \frac{1}{s-1}$

After the terms are entered into this equation, then it is obtained:

$(S^3-1)\, y(s) = \frac{1}{s-1}$

$y(s) = \frac{1}{(s-1)(s^3-1)}$

then taken inverse Laplace transform, then obtained:

$y = L^{-1}[y(s)]$

$= L^{-1}\left[\frac{1}{(s-1)(s^3-1)}\right]$

$= L^{-1}\left[\frac{1}{(s-1)^2(s^2+s+1)}\right]$

$= L^{-1}\left[\frac{\frac{1}{3}}{(s-1)^2}\right] - L^{-1}\left[\frac{\frac{1}{3}}{s-1}\right] + \frac{1}{3} L^{-1}\left[\frac{s+1}{(s^2+s+1)}\right]$

$= \frac{1}{3} x e^x - \frac{1}{3} e^x + \frac{1}{3} L^{-1}\left[\frac{(s+\frac{1}{2}+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{1}{2}\sqrt{3})^2}\right]$

$= \frac{1}{3} x e^x - \frac{1}{3} e^x + \frac{1}{3} e^{½x}(\cos \sqrt[\frac{1}{3}]{3} + \frac{2}{3} \sin \frac{1}{2}\sqrt{3}x)$

So the differential settling is:

$Y = \frac{1}{3}((x-1)e^x + (\cos \frac{1}{2}\sqrt{3}x + \frac{2}{3} \sin \frac{1}{2}\sqrt{3}\, x)e^{-\frac{1}{2}x})$

Example 5

Complete the differential equation:

Y" + y = 8 cos x with the provision of y(0) = 1 and y'(0) =-1

Solution

Taken Laplace transform, then there is a form:

L[y"+y]=8L[cosx]

$S^2 y(s) - s\, y(0) + y(s) = \frac{8s}{s^2+1}$

Once the requirements are entered, then it is obtained:

$(s^2+1)\, y(s) = \frac{8s}{s^2+1} + S - 1$

$y(s) = \frac{8s}{s^2+1} + \frac{S-1}{s^2+1}$

if taken inverse Laplace transform, then there is:

$y = L^{-1}[y(s)]$

$= L^{-1}\left[\frac{8s}{(s^2+1)^2}\right] + L^{-1}\left[\frac{s-1}{s^2+1}\right]$

therefore: $\frac{8s}{(s^2+1)^2} = -4 \frac{d}{ds}\left(\frac{1}{s^2+1}\right)$

Then : $L^{-1}\left[\frac{d^n}{dx^n} f(s)\right] = (-1)^n t^n F(t)$

so =

$Y = L^{-1}\left[4 \frac{d}{ds}\left(\frac{1}{s^2+1}\right)\right] + \cos x - \sin x$

= dx $L^{-1}\left[\frac{1}{s^2+1}\right]$+ cos x – sin x
So the solution of the differential equation is
y = dx cos x + cos x – sin x

Example 6
Determine the completion of the equation:
Y" + uy' + 8y = sin x with the provision of y(0)=1 and y'(0)=0

Solution
Taken Laplace transform, then there is a form
L [y" +uy'+8y] = L[sinx]
[s^2 y(s) – s y(0) – y'(0)]+4[s y(s) - y(0)] + 8 y(s) = $\frac{1}{s^2+1}$

Once the requirements are entered, then it is obtained:
(s^2 + 4s + 8) y(s) = $\frac{1}{s^2+1}$ + s +4
y(s) = $\left[\frac{1}{(s^2+1)(s^2+4s+8)}\right]$+$L^{-1}$ $\left[\frac{s+4}{s^2+4s+8}\right]$

we search first:
$\frac{1}{(s^2+1)(s^2+4s+8)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4s+8}$
1= (AS + B) (s^2+4s+8) + (cs+D) (s^2+1)

From the above form, it is obtained:
A + C=0 ; 4A + B + D = 0 ; 8A + 4B + C = 0 ; 8B + D = 1

Once calculated there are:
A = $-\frac{4}{65}$; B = $\frac{7}{65}$; C = $\frac{4}{65}$; D = $\frac{9}{65}$

Now taken inverse Laplace transform, obtained
(S^2 – 3S +2) Y(S) = $\frac{4}{S^2} + \frac{12}{S+1} + \frac{6S-19}{S^2-35+2}$

Then take inverse Laplace transform:
Y = L^-1[y(s)]
= $L^{-1}\left[\frac{1}{s^2(s^2-35+2)}\right]$ + $L^{-1}\left[\frac{6s-19}{s^2-3s+2}\right]$+ $L^{-1}\left[\frac{12}{(s^2-3s+2)\ (s+1)}\right]$
Y = $L^{-1}\left[\frac{4}{s^2(s-1)(s-2)}\right]$ + $L^{-1}\left[\frac{6s-19}{(s-1)(s-2)}\right]$+ $L^{-1}\left[\frac{12}{(s-1)(s-2)\ (s+1)}\right]$
= (3 + 2x - $4e^{2x}$ + e^{2x}) + (13e^x - 7e^{2x}) + (-6e^x + 4e^{2x} + 2e^{-x})

So solving the differential equation:
$\frac{d^2y}{dx^2} + 4y$ = 9 with the provision of y(0)=0 and y'(0)=7

Solution
When the Laplace transform is taken, it is obtained
S^2 y(s) – s y(0) – y'(0) + 4 y(s) = $\frac{9}{s^2}$

Once the terms are entered, then
(s^2+4) y(s) = $\frac{9}{s^2}$ + 7
y(s) = $\frac{9}{s^2(s^2+4)} + \frac{9}{(s^2+4)}$
taken inverse trasnformasi Laplace, Namely:
Y = L^{-1} [y(s)]
= $L^{-1}\left[\frac{9}{s^2(s^2+4)}\right]$ + $L^{-1}\left[\frac{7}{(s^2+4)}\right]$
= $\frac{9}{4}$ x^2 - $\frac{9}{8}$ sin 2x + $\frac{7}{2}$ sin 2x

So the differential equation is:
y = $\frac{9}{4}$ x^2 + $\frac{19}{8}$ sin 2x

Example 9
Complete the differential equation
y''' – y'' + 3y' – y = x^2 e^x
with the provision of y (0) = 1; y'(0)=0 and y''(0)= - 2

solution

taken Laplace transform, then there are forms:
L[y'''] – 3L[y''] + 3L[y'] – L[y] = L[x^2 e^x]
(s^3 y(s) – s^2 y(0) – s y'(0) – y''(0)) – 3(s^2 y(s) – s y(0) – y'(0)) + 3(s y(s) – y(0)) – y(s) = $\frac{2}{(s-1)^3}$
(s^3 – 3s^2 + 3 s-1) y(s) = $\frac{2}{(s-1)^3}$ + s^2 – 3 s + 1
y(s) = $\frac{2}{(s-1)^6} + \frac{s^2-3s+1}{(s-1)^2}$

to find y taken inverse Laplace transform :
Y = L^{-1}[y(s)]
= $L^{-1}\left[\frac{2}{(s-1)^6}\right] + L^{-1}\left[\frac{s^2-3s+1}{(s-1)^3}\right]$
= $\frac{2x^5e^x x5}{51} + \left[\frac{1}{2}\frac{d^2}{ds^2}(s^2-3s+1) + \frac{d}{ds}(s^2-3s+1).\frac{x^2}{2}\right]_{s=1}^{e^x}$

So the differential solution is:
Y=$e^x - xe^x$-$\frac{1}{2}$ x^2 e^x + $\frac{1}{60}$ + x^5 e^x

V. CONCLUSION

To be able to use the difference calculation with Laplace identity:
1. Through the approach of linear differential equation level n with constant coefficient
2. By entering the principal requirements of differential equations, then daimbil inverse Laplace transform, then will be obtained

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