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Outperforming the Market Portfolio Using Coalitional Game Theory Approach

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Abstract
The view on the balance between risk and return has caused to ignore a part of returns in order to decrease risk. The Game Theory approach focuses on final outcome of investor and maximizing the outcome of player with respect to existing limitations and opportunities which is derived from his/her utility. The optimal answer is not necessarily the best one but it's just the best available answer within the Nash Equilibrium range. In this research, the model of optimal portfolio selection is presented by the cooperative game theory; in the way that players are divided to 5 large groups including: the Major Player (market), Risk-free player, opposite player (contrary to market direction), Risk-averse player and Risky player. It is assumed that the Major Player will have seven strategies for the game, and the other four players, will try to defeat the market through a coalition and a cooperative game. The added value of the cooperation is calculated in terms of a concept called Shapley value and the weights of the optimal portfolio are also calculated using it. The optimal portfolio performance has been evaluated between 2006 and 2017 with respect to the index performance using Sharpe and Treynor criteria. The performance of the proposed cooperative game portfolio was significantly better than the market portfolio for 9 years among 12 examined periods. In 2013, Sharpe's benchmark was better but the Treynor's had a weaker performance than the market, and in general, its preference is ambiguous related to market portfolio. In the years 2010 and 2017, the proposed portfolio of the research was worse than the market portfolio, although the later was not significant based on Jobson and Korkie Statistic. The cooperative game portfolio significantly outperformed the market in all 12 years (2006-2017) based on both Sharpe and Treynor indexes.

Keywords: *Game Theory; Cooperative Game; Shapley value; Portfolio*

I. INTRODUCTION

Financial markets, especially the capital market, are among the most important tools for equipping and allocating financial resources. Considering the strategic financial and economic importance of this market, the financing and allocation of financial resources in the country face a serious problem when there is a significant distortion in it. One of the factors contributing to this, is the lack of an optimal portfolio; in fact, if investors can select their optimal portfolios, while maintaining good performance in financial markets, and encouraging investors to save more, it also increases the level of allocative efficiency and market intelligence, and promotes the most important function of financial markets, which is the optimal allocation of economic resources. If investors do not have the right template to create an optimal portfolio, it not only wastes some of the community resources, but also disappoints actual and potential investors, and with the possible withdrawal of investors from the market, the consequences of a lack of prosperity will hit back the community.

Stocks, fixed income securities and other securities owned by a natural or legal person, constitute an investment portfolio. The basic matter for the problem of selecting a portfolio is the question of what investment instruments and at what rates should be in it. The issue of selecting a portfolio focuses on how much an investor's resources are allocated to each investment tool. The reason for naming this problem as "portfolio selection" in financial literature, is that the investor's objective is not to choose the investment options that are the best one alone, but to choose a portfolio that firstly maximizes returns at a certain level of risk or provide the lowest risk at a certain level of return, and secondly, can perform together in the best way. Various scholars have tried to answer this question with different approaches. An overview of the portfolio selection methods shows that the traditional portfolio selection approach

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was used until after the World War II. In this approach, investors thought that the number of securities in the portfolio must be increased in order to reduce the risk, without paying attention to the return relations among these securities. Various approaches have been proposed to solve the problem of selecting an investment portfolio, which has become popular among scholars by the Harry Markowitz's paper published in a financial journal in 1952, and has a special place in financial literature.

After publishing Harry Markowitz's paper, the modern approach of portfolio selection replaced the traditional approach with focus on the relation between securities. Then Sharpe and later Chen and Ross respectively introduced single-factor and multi-factor models. As forecasted by portfolio management models, investors aimed to reduce risk by diversifying and adding various securities to their portfolios, but because the main strategy was to only reduce the risk, it probably leads to the securities with low returns in the portfolio. The return on portfolio at the end of the investment period cannot be precisely predetermined, since the return on the securities that make up the portfolio is accompanied by uncertainty and sometimes are considered as a random variable. Also, the modern model for the theory of portfolios has assumptions such as the normal distribution of returns, which limits the performance of the model (Kocak, 2014).

By looking at the portfolio selection approaches, we find that although these approaches provide useful information on how to select a portfolio, still there is no public comprehensive approach that can provide an optimal portfolio in different situations, neither in Iran nor in the entire world. There are two basic characteristics in the financial market. The first is the conflict between market players. In financial markets, the total performances of capital market activists or in other word, total games of players, create general performance and direction of market. On the other hand, a player's game is influenced by the overall performance of the market or, in other words, the performance of other players. The second characteristic of financial markets is uncertainty. These two characteristics are the main pillars of game theory, which analyzes the optimal decision of a player. The game theory is a generic term that refers to a set of analytic methods for developing decision-making tools, and evaluates the results of the transaction of individuals with other players using mathematics. Special mathematical techniques have been developed to analyze conflicts in competitive issues. The game theory provides an analytical framework to study the complex transitions between logical players by mathematical tools (Osborne, 2004).

The purpose behind the development of this theory is to assess the rational ways of confronting groups or individuals in economics, engineering, political science, philosophy, and even psychology, and to ensure that one of these players will win (Myerson, 1991). One of the techniques that has recently improved its position among the approaches to the selection of portfolios in the world is the Game theory, but this theory has different types and because of the recent merging of financial science and game theory, there are still many unanswered questions that could be answered using mathematical analysis of game theory. In Iran, as financial science is relatively young, not only using the game theory, but also the general issue of optimal portfolio selection has remained largely neglected. Therefore, the results of this research can increase the motivation, trust and presence of investors in financial markets by optimizing their portfolio in the framework of presenting a template for optimal portfolio selection based on game theory, and also improves information efficiency and allocation efficiency for financial markets and facilitates the transfer of public resources to productive activities.

This theory divides games into two broad categories; cooperative games and non-cooperative games. Cooperative games are a kind of game in which the players join each other to obtain more utility. Given the fact that investors involved in a large financial market are playing in a large game whose utility are dependent on the other investors and the market, forming a cooperation between investors can increase the collective utility of them. Therefore, the cooperative game approach can provide a solution to find optimal portfolios. Therefore, the main problem of the research is to provide a model for optimizing the investment portfolio using game theory.

II. LITERATURE REVIEW

The game theory attempts to mathematically evaluate behavior in a strategic condition or in a game, in which the success of a player in his choice depends on the selection of other players. In other words, a game is formed between the two or more entities if the profit of an entity does not depend solely on its own behavior and is influenced by the behavior of one or more other entities, and other decisions have a positive or negative effect on its profit. Games have many dimensions and there are different classes for it (Abdoli, 2016);

A) Static or Dynamic game; in a static game, the players play simultaneously, and none of them will know what the opposing player does. But in the dynamic game, moves of players will be turn-based. For example, participating in a single-stage auction is a static game, because, until the time of opening the proposed envelopes, none of the players knows the strategy of QBA, but in the opposite, chess game is dynamic. In a static game, the player has to guess which strategy the player chooses to adopt, but in the dynamic game, each actor must also consider the consequences of his chosen strategy.

B) Conflict of interest or possibility of cooperation; in most games, the amount of a player's gain is exactly the amount of opponents' losses, which is called Zero-sum game. In other words, the sum of gains and losses of the players will be equal to zero. There is a complete conflict of interest in such games. While sometime, despite the fact that the interests of the players may not be the same with each other, there is a situation where they can work together for more resources. However, once the result is shared, the conflicts of interest are again revealed.

C) Cooperative or non-cooperative games; Games can be "Cooperative" or "Non-cooperative". In non-cooperative games, players may eventually choose the strategy that does not have the most benefit possible in order to maximize their outcome depending on the opponents' selective strategies; hence, in the cooperative game, the players try to gain more benefit by cooperation with other players.

Looking at the mechanism of the capital market, it can be seen that the players in this market are players of a static game with a conflict of interest, so this research tries to find an optimal solution for portfolio selection, by explaining the behavior of investors in the framework of a cooperative game. Therefore, the model of this research will be the zero-sum (between investors and the market), and a cooperative game (among cooperators) in a static game model.
Basically, there are two main criteria in the game; 1) the number of players in the game; 2) the net result of the game. The first criterion is only the participants of the game with a conflict of interest. The second criterion makes it possible to distinguish between zero-sum games and games with non-zero summation. A zero-sum game is a kind of game that the positive outcome of participants is assumed as the same negative outcome for other players.

The game theory is based on the rational comparison of the participant's behavior and allows the possibility of achieving a balance in these situations. The first player is afraid that the second one will recognize his chosen strategy and it will be easy for it to predict the behavior. This theory tries to analyze the use of a mathematical technique that addresses contradictory situations. This model was first introduced by Borel in 1921 and was developed by Von Neuman and Morgenstern (Asgharpour, 2014).

If the player1 and player2 has m and n strategies respectively, the probabilistic consequences of the game can be shown by the profit and loss matrix;

Equation 1

$$pay\ off = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m,1} & a_{m,2} & \cdots & a_{m,n} \end{bmatrix}$$

where a_{ij} represents the gain of player1, if player1 and player2 adopt strategy i and strategy j respectively. In a zero-sum game, the gain of player1 would be the loss of the player2 and vice versa, so the minimum profit for each player is the maximum expected gain of the competitor. If player1 choose strategy i, the minimum profit would be $\min_j a_{ij}$ which is the lowest expected profit in second row of matrix. Here, the player1 tries to adopt a strategy to turn the minimum profit to maximum. In other words, the strategy is better to have least negative outcome. Thus, the expected outcome is $\max_i \min_j a_{ij}$. By this strategy, the outcome of the game might be better but not worse. In contrast, the second player fears that the player1 is aware of his/her information and behavior, and if he chooses the j_{th} strategy, the largest member of the j column which is the maximum profit of the first player, would be the minimum profit of the second player and the player2 choose a strategy in a way that will minimize the maximum profit. Therefore, his/her strategy outcome will be $\min_j \max_i a_{ij}$.The balance between two players' decisions will exist when below equation is true.

Equation 2
$$\min_j \max_i a_{ij} = \max_i \min_j a_{ij}$$

This state is called Nash Equilibrium. In game theory, the Nash equilibrium (John Forbes Nash) is a game theory solution consisting of two or more players, which assumes that each player is aware of the player's balance strategy, and without any player who only tries to change his profit by unilateral strategy change. If any player chooses a strategy, no player can act by changing his strategy while maintaining the interest of the other player unchanged, then the set of current strategy choices and the relevant benefit, forms the Nash equilibrium. It should be noted that Nash equilibrium does not necessarily have the best overall productivity for all players. In most games, a player need a strategy that is not expected by the rival player. In such games, it's obvious that no player wants a rival player to accurately anticipate his/her behavior. Thus, the player1 selects strategy i with probability of p_i, and the player2 chooses strategy j with probability of q_j. Therefore, the outcome of the first player's strategy will be equal to the least mathematical expected benefit in choosing any strategy given the chances of choosing a strategy by the competitor player;

Equation 3
$$v = \underset{i}{Min}\left(\sum_{i=1}^{n} a_{i,1}p_{i,1}, \sum_{i=1}^{n} a_{i,2}p_{i,2}, \dots, \sum_{i=1}^{n} a_{i,m}p_{i,m}\right)$$

and the player1 chooses his/her strategy by maximizing v as following:

Equation 4
$$Max\ v;\quad st:\ \sum_{i=1}^{n} p_i = 1 \quad , \quad p_i \geq 0 \quad \forall i$$

Conversely, the outcome of the second player's selected strategy is u, and the player seeks to minimize it.

Equation 5
$$u = \underset{i}{Max}\left(\sum_{j=1}^{m} a_{1,j}q_{1,j}, \sum_{j=1}^{m} a_{2,j}q_{2,j}, \dots, \sum_{j=1}^{m} a_{n,j}q_{n,j}\right)$$

Equation 6
$$Min\ u;\quad st:\ \sum_{j=1}^{m} q_j = 1 \quad , \quad q_j \geq 0 \quad \forall j$$

In a Nash equilibrium state, both players adopt their strategy with the assumption of the balance between players, so the overall equilibrium of a game will be determined by the following optimization model.

Equation 7
$$\begin{aligned} & Max\ (v-u) \\ & st: \\ & u \leq \sum_{j=1}^{m} a_{1,j}q_{1,j} \qquad \forall i \\ & v \leq \sum_{i=1}^{n} a_{i,1}p_{i,1} \qquad \forall j \\ & \sum_{i=1}^{n} p_i = 1, \sum_{j=1}^{m} q_j = 1 \\ & p_i \geq 0 \ \ \forall i, \qquad q_j \geq 0 \ \ \forall j \end{aligned}$$

By calculating Nash equilibrium, and the probability of adopting any p_i strategy and any q_j strategy by rival, the value of the game will be calculated for each player.

Equation 8
$$v(s) = \sum_{i=1}^{n}\sum_{j=1}^{m} p_i^* q_j^* a_{ij}$$

A. Mathematical Model of Cooperative Game

In the cooperative game theories, the main focus is on creating possible cooperation and the added benefit obtained by all players through this cooperation. In a general game with n players, each sub-set (except the empty set) for a set of players can be considered as a cooperation. In each game with a hypothetical cooperation "s", one can define $v(s)$, which is called a characteristic function. Indeed, for each $S \subset N$ cooperations, one can define $v(s)$, which represents the total utility that the cooperation members gain from working together, no matter what strategies are adopted by players outside the cooperation. It can be said that in the game, S cooperation participants play non-cooperative game with other players that are not in the cooperation. Standard cooperative games are cooperative games that will lead to the most possible utility. In other words, the cooperation games that have these two general characteristics are called Canonical Game Theory:

- The game must have a special state that is defined as $2^N \to \mathbb{R}$
- Include Superadditivity in cooperation

In a cooperation, players will gain the same benefit that they would gain if it was a non-cooperative game; i.e. the cooperation of players will cause superadditivity. Superadditivity is defined as below:

Equation 9
$$v(S_1, S_2) \supset \left\{x \in \mathbb{R}^{|S_1 \cup S_2|} / (x_i)_{i\in S_1} \in v(s_1), \left(x_j\right)_{j\in S_1} \in v(s_2)\right\} \quad \forall\, S_1 \subset N, \quad \forall\, S_2 \subset N, \quad S_1 \cap S_2 = \emptyset$$

Equation 10
$$v(S_1, S_2) > v(S_2) + v(S_1) \quad \forall\, S_1 \subset N, \quad \forall\, S_2 \subset N, \quad S_1 \cap S_2 = \emptyset$$

In order to allocate the added utility or benefit, the concept of Shapley Value is presented. The Shapley Value for each cooperator in an n-player game is calculated using added value of each cooperation and the possibility of it, as follows:

Equation 11
$$\varphi_i(v) = \sum_{i\in S} \frac{(n - |S|)!\,(|S| - 1)!}{n!} \left[v(S) - v(S\backslash\{i\})\right] \qquad i \in S$$

$\varphi_i(v)$ is the Shapley Value of player i, n is the total number of players, s is number of players in cooperation, $v(s)$ is the value of a cooperative game, and $v(s\{i\})$ is cooperative game value without player i. In fact, Shapley Value, calculates the added value of player i provided entering to each cooperation with respect to the probability of entering the player to cooperation. It's notably that the total Shapley value is called as Characteristic Function.

Equation 12
$$v(N) = \sum_{i=1}^{N} \varphi_i(v)$$

In the portfolio optimization model, the Shapley value can determine the weight of each cooperation and be the basis for calculating the weight of each security in the optimal portfolio. The traditional theory of portfolio was accepted until after World War II (Shenoy & McCarthy, 1998). In this approach, investors thought that the number of securities in the portfolio should be increased in order to reduce the risk, without paying attention to the return on equity (Reilly & Brown, 1999). In order to overcome uncertainty, Harry Markowitz published his paper titled "Portfolio Selection" and it led to significant development in this area. The publication of his article initiated the modern portfolio theory.

According to this theory, the increase in the number of stocks in the portfolio is not enough alone, but the intensity and direction of the relationship between securities in the portfolio is also effective (Markowitz, 1952).

Sharpe (1971) introduced a single-index model and attempted to explain stock returns by a factor that is called market index. Chen and Ross (1986) introduced multi-factor models. This model is based on the assumption that the stock returns is dependent to a number of economic variables, in addition to the market index, including interest rates and industry index (Elton & Gruber, 1995). Konno and Yamazaki (1991) also developed a new model for selecting a portfolio based on average-variance (Konno & Yamazaki, 1991).

The Cooperative game has found a special place among the various majors that need coordination, such as communication networks (Saad, Han, Debbah, Hjourngnes, & Başar, 2013). In particular, wireless networks are kinds of cooperation, whose fair distribution is very important. In his study, Saad et al. described the game theory in game theory in three ways and proposed a model for connecting wireless networks based on needs of engineers. Cohen et al. Also used the cooperative game theory to feature selection (Cohen & Vijverberg, 2008). Lemaris examined five fundamental applications of game theory in the insurance industry (Lemaris, 2013). Kocak (2014) has also taken a look at the selection of optimal portfolio using the game theory on the London Stock Exchange.

III. RESEARCH METHODOLOGY

In this research, we intend to divide the market players into five groups:

- Big player: The big player is actually the market index. It has the power to shape the market, and other market players are trying to defeat the big player by the cooperation with each other.
- Risk-free player: Risk-free player does not want to accept any risk. Therefore, the investor will invest heavily in risk-free securities, and his gains will be risk-free returns.
- Opposite player: The Opposite player is interested in moving in the opposite direction of the big player. In the financial language, the Opposite player only invests in securities with negative beta.
- Risk-averse player (low risk): A low-risk player in stocks and securities with a beta less than 1.
- Risky player (high risk): A high-risk player in stocks and securities with a beta more than 1.

It is assumed that the big player will have 7 strategies to play. Each of the securities is considered as one of the strategies for one of the other players based on the beta criterion. For example, portfolios A and B with betas 1.2 and 1.5 respectively, would be risky player's strategies. Assuming that a high-risk player has just these two strategies, his gain table has six modes, each of which will highlight the returns of a high-risk player strategy compared to the big player's strategy. Each player has a gain (utility) individually. Obviously, players will only participate in a cooperation if the utility and gain from the cooperation are equal or more than that for individual player or other cooperations that the player can join in. The probability of forming each cooperation is determined by the number of participants in the cooperation divided by the total number of players. Then the utility of each player will be calculated using the probability of forming the cooperation and the expected gain. Finally, the total weight and the strategy of each player will be calculated based on the model used in the research. The total weight of each player is the amount of portfolio assigned to risk-free securities, negative-beta securities, low risk securities and risky securities, and the weight of each strategy represents the weight of each security in the optimal portfolio. Given the fact that market players have been able to achieve a higher utility by using a cooperation rather than an individual game, the portfolio will be optimal in this state.

The utility of each player will be calculated using three criteria, which are calculated based on below notes:

- If the return of the player's strategy exceeds the return of the big player (market), then it will be 1 unit of positive utility, and otherwise it will be 1 unit of negative utility.
- If the return of the player's strategy is more than the return of the risk-free securities, it will be 1 unit of positive utility and otherwise it will be 1 unit of negative utility.

- If the risk-adjusted return (Sharpe ratio) of the player's strategy is greater than the big (market) player's Sharpe ratio, it will be 1 unit of positive utility and otherwise it will be 1 unit of negative utility.

The overall utility is the total score of three criteria above. The equation of utility calculation will be as follows:

Equation 13
$$Utility = \begin{cases} \forall\, r_i > r_m : U_1 = 1, & \forall\, r_i < r_m : U_1 = -1 \\ \forall\, r_i > r_f : U_2 = 1, & \forall\, r_i < r_f : U_2 = -1 \\ \forall\, SR_i > SR_m : U_3 = 1, & \forall\, SR_i < SR_m : U_3 = -1 \end{cases} ;\; U_i = U_1 + U_2 + U_3 + 3$$

It is worth mentioning that in order to avoid the negative utility, the number three has been added to the score. The desirability of a hypothetical cooperation S is the sum of its players' utilities.

Equation 14
$$U_s = \sum_{i \in S} U_i$$

U_s as the gain or utility of each cooperation, is the raw material of the Pay_Off matrix (Equation 1), and will enter Model 6 to calculate the optimal solution for the cooperation. Given that four players have been defined in this study for cooperation against the big player (including the opposite player, a non-risk player, risk-averse player, and risky player), the study was conducted with 15 cooperations, and consequently 15 problems ($2^4 - 1$) to calculate an optimal solution.

By calculating the optimal solution, using Equation 7, the value of each of these 16 cooperations (v(s)) is obtained, which represent the overall utility in the cooperative game for contributors. A cooperation that creates the highest value will be the basis for a portfolio optimization model. Given the superadditivity value and equations 8 and 9, it is evident that the participation of all players in the cooperation leads to the highest value of the game and the utility v(s).

In the next step, the extra value of four defined players will be calculated using the Shapely value method (Equation 11). As noted above, the Shapely value of player i shows that the entry of the player to each of the 16 cooperations will, on average, increase the utility, by considering the possibility of joining the alliances. This index shows the bargaining value of each player to enter into the cooperation. The final result of these calculations is the Shapley vector.

Equation 15
$$Shapley\ Vector:\ \varphi = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)$$

We can calculate the weight of each player or the bargaining power of each player, by standardization of Shapley vector. It should be noted that the weight of each player P_i is calculated by dividing the Shapley value of the player into the characteristic function (Equation 11).

The possibility of occurring strategies that investment tools of each players number 1, 2, 3 and 4 are selected by them, are defined as α_j^*, β_k^*, γ_h^* and δ from the solution that present optimal solution v(s) S={1,2,3,4}. δ is the risk-free player, α is the opposite player (j=1,2,…,n), β is the risk-averse player (k=1,2,…,m) and γ is the risky-taking player. The risk-free player only has one investment tool.
The investment percentage of instrument j in the investment options of player α in the total portfolio is equal to the following:

Equation 16
$$W_j = p(\alpha)\, \alpha_j^* \sum_{k,h} \beta_k^* \gamma_h^* \delta$$

Also, the percentage of instrument k in the investment options of the player β and the investment instrument k in the investment options of player γ, and the percentage of the risk-free investment (δ) in the total investment portfolio is calculated based on below equations:

Equation 17
$$W_k = p(\beta)\, \beta_k^* \sum_{j,h} \alpha_j^* \, \gamma_h^* \, \delta$$

Equation 18
$$W_h = p(\gamma)\, \gamma_h^* \sum_{k,j} \alpha_j^* \, \beta_k^* \, \delta$$

Equation 19
$$W_\delta = p(\delta)\, \delta \sum_{k,j,h} \alpha_j^* \, \beta_k^* \, \gamma_h^* \xrightarrow{\sum_{k,j,h} \alpha_j^* \beta_k^* \gamma_h^* = 1} W_\delta = p(\delta)$$

After calculating the portfolio of assets, its function is compared to the market index performance based on the Treynor criteria. Treynor index is calculated as follows:

Equation 20
$$T_p = \frac{R_p - R_f}{\beta_p}$$

After clarifying the overall research space, we will now list the exact details of this study. The society data and statistical sample are evaluated for the years 2006 to 2017 and the expected returns are calculated, along with the predicted Sharpe ratio at the beginning of each year. Then, by optimizing the portfolio using game theory, and in particular the Canonical Game Theory, we will get to the expected portfolio. The results of the optimized portfolio are compared for each year to the total market index efficiency for the same year, and finally the optimal portfolio performance based on game theory is compared with the performance of the market index.

In terms of inferential statistics, the difference between the two indexes for the two samples should be statistically significant. That is, if an index for sample 1 is greater than that index in sample 2, then the randomness of the difference or the equivalence hypothesis must be rejected. The difference in the performance of two portfolios calculated by the Sharpe index should also be statistically evaluated. In order to determine the superiority of a portfolio performance from another, the meaning of the difference in the criteria for evaluating its performance should be demonstrated. In this regard, Jobson and Korkie (1981) introduced a statistic for significance analysis of the difference between Sharpe and Treynor indexes. The statistic for the Sharpe ratio is calculated as equation 21:

Equation 21
$$z = \frac{\sigma_1(\mu_2) - \sigma_2(\mu_1)}{\sqrt{\theta}}$$

Equation 22
$$\theta = \frac{1}{T}\left(2\sigma_1^2\sigma_2^2 - 2\sigma_1\sigma_2\sigma_{1,2} + \frac{1}{2}\mu_1^2\sigma_2^2 + \frac{1}{2}\mu_2^2\sigma_1^2 - \frac{\mu_1\mu_2}{2\sigma_1\sigma_2}\left(\sigma_{1,2}^2 + \sigma_1^2\sigma_2^2\right)\right)$$

Based on this assumption, the null hypothesis is smaller or equal Sharpe ratio in the traditional and classic portfolio selection method, and the hypothesis claims that the proposed strategy has a better performance. The z statistic follows a normal distribution (Jobson & Korkie, 1981).

$$\begin{cases} H_0: SR_1 - SR_2 = 0 \\ H_1: SR_1 - SR_2 \neq 0 \end{cases}$$

The statistic for the Treynor ratio is calculated as equation 23:

Equation 23
$$Z_{Tr} = \frac{\frac{\sigma_{1,m}}{\sigma_m^2}(\mu_2) - \frac{\sigma_{2,m}}{\sigma_m^2}(\mu_1)}{\sqrt{\psi}}$$

Equation 24
$$\psi = \frac{1}{T}\Big(\sigma_1^2\sigma_{2,m}^2 + \sigma_2^2\sigma_{1,m}^2 - 2\sigma_{1,m}\sigma_{2,m}\sigma_{1,2} + \mu_1^2\left(\sigma_m^2\sigma_2^2 - \sigma_{2,m}^2\right) + \mu_2^2\left(\sigma_m^2\sigma_1^2 - \sigma_{1,m}^2\right) - 2\mu_1\mu_2\left(\sigma_m^2\sigma_{1,2} - \sigma_{1,m}\sigma_{2,m}\right)\Big)$$

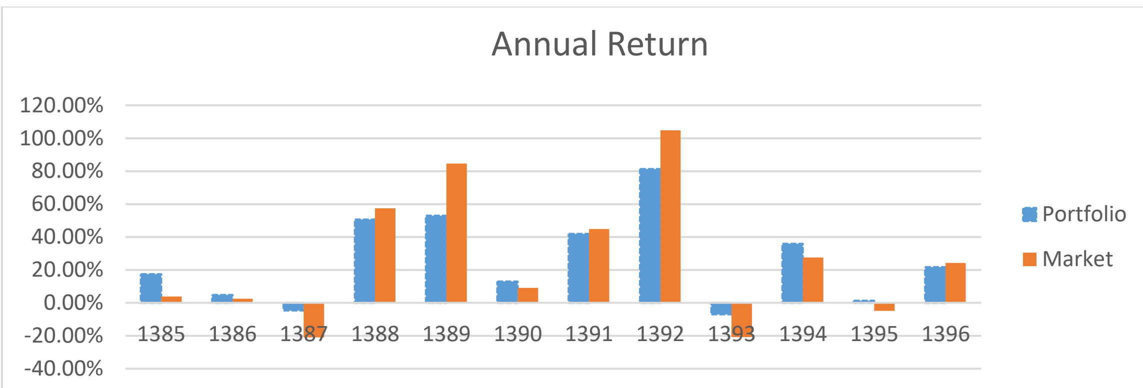
Based on this assumption, the null hypothesis is smaller or equal Sharpe ratio in the traditional and classic portfolio selection method, and the hypothesis claims that the performances are different in portfolios. The z statistic follows a normal distribution.

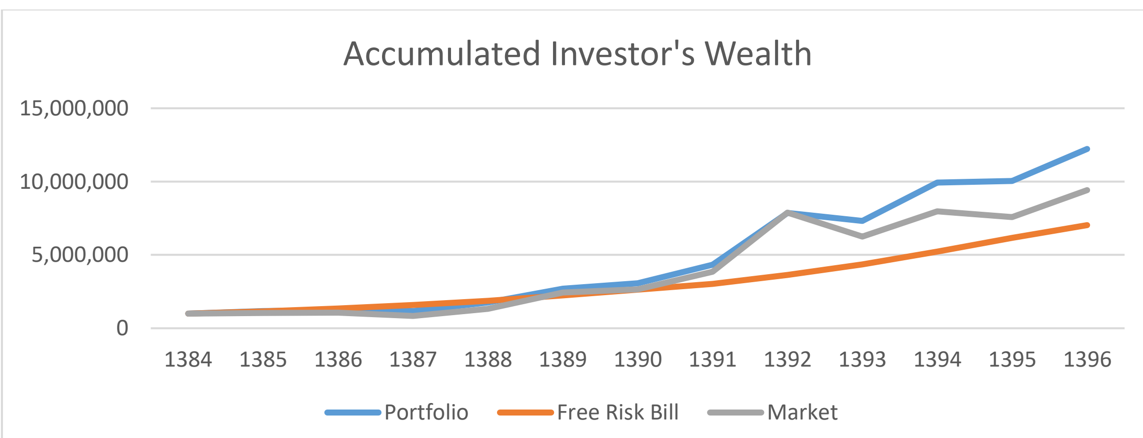
$$\begin{cases} H_0: Tr_1 - Tr_2 = 0 \\ H_1: Tr_1 - Tr_2 \neq 0 \end{cases}$$

Methods of data analysis are described in detail in the research methodology. Analysis tools also include various statistical softwares that are able analyze these data. Data analysis tool in this study is MATLAB software.

IV. FINDINGS

In order to calculate the optimal portfolio, first, each player's investment strategies and its utility is calculated, then, based on a Zero-Sum Game solution, the shapely value is calculated based on the value of each player in the cooperation, and the optimal portfolio weights have been calculated by standardizing the shapely value. After selecting the optimal portfolio by using the various market players in the cooperation against the big player (The INDEX), the performance of the proposed portfolio must be compared with the performance of the total market index. In the case of the superiority of the proposed portfolio and the greater return on risk, it can be argued that the proposed portfolio is able to defeat the market using game theory. Total index return is obtained by dividing index number at the end of the year on the index number at the beginning of the year, and daily deviations were used to calculate the standard deviation; so that the daily deviation of the total index is multiplied in the number of working days of the year to calculate annual standard deviation of the index return. The return and standard deviation of the portfolio are also calculated in the same way as the total market index. The summary of the results of the proposed portfolio and the total market index is as follows:





In the following, we compare the performance (taking into account the risk and returns simultaneously) of the proposed portfolio based on the game theory and the performance of the total market index as a market agent. The Treynor Index is an indicator that modifies the excess returns to the risk-free rate based on systematic risk and evaluates the performance of the two portfolios (in this study, the proposed portfolio based on the game theory and the total market index). The Sharpe index also adjusts the additional reward on risk-free returns based on the standard deviation of return (the criterion of total investment risk) and evaluates the performance of the two portfolios based on overall risk. Sharpe and Treynor performance evaluation criteria for the proposed portfolio of game theory and the total market index are as follows:

year	Portfolio		Market		Portfolio's Performance relative to Market(Treynor (Ratio	Portfolio's Performance relative to Market(Sharpe (Ratio	Overal Result
	Sharpe Ratio	Treynor Ratio	Sharpe Ratio	Treynor Ratio			
1385	**0.2538**	**0.0171**	**2.5055-**	**0.1222-**	**Winner**	**Winner**	**Winner**
1386	2.1431-	0.1152-	2.0539-	0.1346-	**Winner**	**Winner**	**Winner**
1387	5.0395-	0.2387-	3.7506-	0.3901-	**Winner**	**Winner**	**Winner**
1388	5.8233	1.2784	4.1541	0.3939	**Winner**	**Winner**	**Winner**
1389	4.6958	0.3962	6.3247	0.6468	**Looser**	**Looser**	**Looser**
1390	0.4522-	0.0413-	0.7054-	0.0795-	**Winner**	**Winner**	**Winner**
1391	2.6141	0.3152	2.4003	0.2975	**Winner**	**Winner**	**Winner**
1392	6.0268	0.7666	5.6174	0.8469	**Looser**	**Winner**	**Ambiguous**
1393	4.2129-	0.3149-	4.0465-	0.4086-	**Winner**	**Winner**	**Winner**
1394	1.4025	0.5105	0.6546	0.0757	**Winner**	**Winner**	**Winner**
1395	3.0611-	0.2068-	3.4921-	0.2289-	**Winner**	**Winner**	**Winner**
1396	1.5665	0.0927	1.8428	0.1027	**Looser**	**Looser**	**Looser**

In order to evaluate the performance of two portfolios based on the Sharpe ratio and Treynor index, if the index number is positive, the higher index represents better performance. Also, if one of indexes is positive and the other is negative, the positive one has a better performance. If both portfolios have returns less than zero-risk returns, the Sharpe index number and possibly the Treynor index (if the portfolio beta is not negative) has a negative number. In this case, the interpretation and evaluation of the portfolio performance based on these two indexes, requires a closer examination of risk and returns. For example, in 2007, the proposed portfolio of game theory provided 4.86 percent and the total market index had 2.54 percent, both lower than the risk-free return rate (16 percent) in the same year. Although this year, the Sharpe index number of the proposed portfolio is smaller (more negative) than the Sharpe total market index number, the return of proposed portfolio is more than market return and its standard deviation is smaller than the standard deviation of the market. It can be concluded, therefore, that the portfolio has defeated the market in this year based on the Sharpe ratio.

Also, in interpreting the performance evaluation criteria, it is likely that Sharpe and Treynor criteria differ from each other. In this case, it is not possible to say with certainty that the performance of a portfolio is better than the other. For example, in 2013, the proposed portfolio of game theory has performed better than the market in terms of Sharpe benchmark, however the market has failed based on Treynor so general assessment of the performance for this year is ambiguous.

According to the above description and as it can be seen in the table above, based on the Sharpe ratio, the proposed portfolio of game theory has performed better than the total market index in all years studied, except for 2010 and

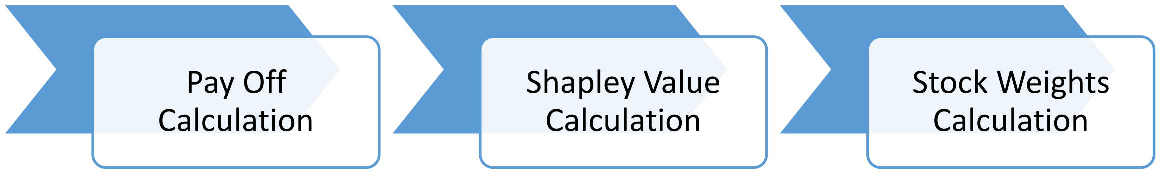
2017. In terms of the Treynor criterion, the proposed portfolio of game theory has only lost in 2010, 2013, and 2017. In a general conclusion, it can be said that the performance of the proposed portfolio is defeated in 2010 and 2017, and in other years, except in 2013, it has beaten the performance of the total market index. The 2013 performance evaluation is not clear because of different rates of Sharpe and Treynor indexes.

Year	Performance Portfolio's relative to (Market(Treynor Ratio	Portfolio's Performance relative to Market(Sharpe (Ratio	Overal Result	Statisticaly meaningful(SM) %95in Confidence
1385	Winner	Winner	Winner	SM
1386	Winner	Winner	Winner	SM
1387	Winner	Winner	Winner	SM
1388	Winner	Winner	Winner	SM
1389	Looser	Looser	Looser	SM
1390	Winner	Winner	Winner	SM
1391	Winner	Winner	Winner	SM
1392	Looser	Winner	Ambiguous	SM
1393	Winner	Winner	Winner	SM
1394	Winner	Winner	Winner	SM
1395	Winner	Winner	Winner	SM
1396	Looser	Looser	Looser	Not Reject The Null Hyp
Entire Period	Winner	Winner	Winner	SM

According to Jabson Korkie's test results for the research hypotheses, almost all of the model's victory and failure results were confirmed statistically. However, just the hypothesis about lower performance of the suggested portfolio based on Sharpe benchmark compared to the performance of the total market index in 2017 was statistically insignificant. Considering the accepted research hypotheses that the proposed portfolio of game theory is superior to the total market index in terms of the Sharpe and Treynor criteria, the main hypothesis of the study that the proposed portfolio of games theory is optimal and provides higher returns at a certain level, can be approved.

CONCLUSION AND DISCUSSION

The decision making problems have been the issue of human mind for many years. This has sometimes been solved through trial and error, and sometimes by the invention of some techniques. Game theory is an approach that offers positive and promising results to rational individuals. Attempting to select a portfolio requires the application of new techniques and has been able to fulfill the human goals to a large extent. The greater return of the portfolio based on cooperative game theory, compared to market return and the risk-free rate as well as the superiority of this portfolio based on Treynor index, indicates the success of the model in portfolio selection.



The research process is in the form of the above exhibition. Based on the findings of this paper, the proposed portfolio using the cooperative game theory had better performance in most of the examined periods and in general, compared to the total market index. Defeating the market in active management while believing market efficiency has been a long-standing human capital investment issue that can be solved by defining a canonical cooperative game. The main point here is that the zero-sum game has been used in solving this game in which we will have at least one winner and at least one loser, and the sum of all players' gains will be zero. As a result, it can be predicted that if all market players simultaneously use this template and with the same data, the probability of success of the template will be reduced.

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