Dama International Journal of Researchers ISSN: 2343-6743, Scientific Journal Impact Factor: 5.968 & ISI Impact Factor: 1.018,
Dama Academia Pubisher: Vol 3, Issue 06, June, 2018, Pages 41- 83, Available @ www.damaacademia.com

KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

PERFORMANCE MEASURE OF VALUE AT RISK USING MONTE CARLO APPROACH AND HISTORICAL SIMULATION

OSEI JONES
(Bed. Mathematics, TR's Certificate 'A')

A THESIS SUBMITTED TO THE DEPARTMENT OF MATHEMATICS, KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI, IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF PHILOSOPHY IN APPLIED MATHEMATICS (MPHIL.)

OCTOBER, 2016

Declaration

I hereby declare that this submission is my own work towards the award of the M. Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgment had been made in the text.

OSEI JONES	....	....
STUDENT	SIGNATURE	DATE

CERTIFIED BY:

DR. J. ACKORA-PRAH	....	....
SUPERVISOR	SIGNATURE	DATE

CERTIFIED BY:

DR. R .K. AVUGLAH	....	....
HEAD OF DEPARTMENT	SIGNATURE	DATE

Dedication

I dedicate this thesis to my children Federica Osei Amoah, Moses Osei Ladijo and Ethel Nhyira Osei for their unfailing emotional support and understanding during my study.

Acknowledgements

First and foremost, I must acknowledge and thank God for protecting and guiding me throughout this period of study. I never have accomplished this without the faith I have in God.

I would like to express my profound gratitude to my supervisor, Dr. Joseph Ackora-Prah and Peter Kwasi Sarpong for his support, encouragement, supervision and useful suggestions throughout this study. His moral support and continuous guidance enabled me to complete my work successfully. I am also highly thankful to my friends and study group mates especially Asante for their valuable suggestions throughout this study. Lastly, to all those who contributed in many diverse ways towards this thesis I say God richly bless you.

Abstract

Value at risk (*VaR*) is a management tool for measuring and controlling risk. Individual and institutional investors rely their investment decisions increasingly on the risk inherent in a security. In this theses, calculating of *VaR* are implemented using Historical Simulation and Monte Carlo approach on stock portfolio. Different Values of confidence levels are also used for each of the method. The study is conducted on six fundamentally different stocks. Data on daily prices on collected for a period of eight years (2007-2014) for all stocks assets and their corresponding log returns calculated.

From our analysis, Monte Carlo Simulation had an optimal values of *VaR* as compared to Historical simulation in both the *VaR* 95% and *VaR* 99% confidence levels. Nonetheless, the *VaR* 95% has the highest simulation time.

Table of Contents

Chapter 1
Introduction

1.1 Background

Researchers in the field of financial Mathematics and Economics have long identified the significance of measuring the risk of a portfolio of financial assets or securities. Vehemently, concerns go back at least forty years, when Markowitz's earth shattering work on portfolio choice (1959) investigated the suitable definition and estimation of danger. In the field of investment, risk is a measure of how unstable assets returns are. Introduction to this instability can prompt a misfortune in one's ventures. Thus instruments are utilized not just to latently measure and report risk, but to protectively control or effectively oversee it. Notwithstanding, a system progressed in writing includes the utilization of Value-at-Risk (VaR) models. The idea and utilization of value at risk is recent. Value at risk was first utilized by major financial firms in the late 1980's to quantify the dangers of their exchanging portfolios. Right now Value at risk was utilized by most major derivative dealers in remote nations to gauge and oversee market hazard. It is additionally progressively being utilized by smaller financial organizations, non-financial organizations, and institutional investors. A *VaR* model measures market risk by deciding how much the estimation of a portfolio could decrease over a given timeframe with a given likelihood as an aftereffect of changes in interest rates, foreign exchange rates, equity prices, or commodity prices. For instance, if the given timeframe is one day and the given likelihood is 1 percent, the *VaR* measure would be an evaluation of the decrease in the portfolio esteem that could happen with a 1% likelihood throughout the following exchanging day. At the end of the day, if the *VaR* measure is exact, losses greater than the *VaR* measure ought to happen under 1% of the time. *VaR* models total the several components of price risk into a single quantitative measure of the potential for losses over a specified time horizon. These models are obviously engaging in light of the fact that they pass available risk of the whole portfolio in one number.

There are diverse procedures to compute the *VaR*, most popular are Historical simulation, Monte-Carlo simulation. Variance-Covariance, J. P. Morgan's Risk Metrics System. For investors, danger is about the chances of losing cash, and VaR depends on that common-sense fact. By accepting that investors think about the chances of enormous misfortunes, VaR can be utilized to answer the inquiries. The *VaR* measurement has three parts: a period, a certainty level and a loss amount (or loss percentage). It can thus be used to answer question such as: “What is the most I can (with a 95% or 99% level of confidence) expect to lose in Cedis over the next month”?, ”What is the maximum percentage I can (with 95% or 99% confidence) expect to lose over the next year”?, ”What is my worst-case scenario"? or ''How much could I lose in a really bad month"?

1.2 Statement of the Problem

Since the publication of JP Morgan's Risk Metrics in 1994 there has been an explosion in the research in the areas of value of risk and risk management in general. While the fundamental ideas encompassing *VaR* are founded in the area of market risk measurement they have been reached out throughout the most recent decade, to different territories of risk management. Specifically, *VaR* models are presently usually used to gauge both credit and operational risks.

In any case, with different methods and models, the decision that *VaR* users face is the decision of selecting the proper procedure that is generally suitable. The strategies ought to make gauges that fit the normal conveyance of returns. On the off chance that *VaR* is overestimated, the administrators winds up overestimating the danger. This, in any case, could bring about the holding of great measures of cash to cover misfortunes as for the situation with banks under the Basel II accord, (Basle Committee on Banking Supervision, 1996). The same goes for the inverse occasion, when *VaR* has been thought little of bringing about inability to cover acquired losses.

1.3 Objective of the Research

This study seeks to compare two different methods to calculating *VaR* namely Historical Simulation and Monte Carlo Simulation. The method will be applied on six different equities on the Ghana Stock Exchange Market with two different confidence level of 95% and

99%.

1.1 Significance of the Study

The study will be significant in the following ways:

1. When managed properly, *VaR* can provide a controlled way of getting high returns on ones investments.
2. It will assist the manager to know the type of approach suitable in calculating *VaR*.

1.2 Methodology

In this thesis, we will be calculating and evaluating *VaR* for six (6) different stocks on the Ghana Stock Exchange (GSE) market and the results will be evaluated. Two different techniques (Historical simulation and Monte Carlo simulation) will be used in our evaluation with different confidence levels of 99% and 95%. Simulation will be carried out using (MATLAB) software and results graphed and analyzed.

1.3 Thesis Outline

This thesis contains five (5) main chapters. Chapter one (1) examines the foundation of the study, the problem statement, Objectives and also Significance of the research, the methodology. Chapter two (2) investigates the reasons for picking Value-at-Risk as a research study, limitations of Value at risk, and some previous research relevant to the study.

Fundamental definitions applicable to our study, the technique utilized as a part of completing this study, and the tackling steps are clarified in Chapter three (3). Investigation and results are investigated in Chapter four (4). Conclusion are discussed in Chapter five (5)

Chapter 2

Literature Review

2.1 Introduction

Risk estimation has engrossed financial market participants since the beginning of financial history. Be that as it may, numerous past endeavors have turned out to be unreasonably perplexing. For instance, upon its presentation, Harry Markowitz's Nobel prize-winning theory of portfolio risk measurement was not embraced practically speaking as a result of its burdensome information prerequisites. In fact, it was Sharpe (2000) who, alongside others, made portfolio theory the standard of financial risk measurement in real world applications through the appropriation of the rearranging suspicion that all risk could be decomposed into two parts: systematic, market risk and the residual, company-specific or idiosyncratic risk. The resulting Capital Asset Pricing Model theorized that since just undiversifiable market risk is relevant for securities pricing, only the market risk measurement β is necessary, along these lines extensively decreasing the required information inputs, (Sharpe, 2000). This model yielded a promptly quantifiable evaluation of risk that could be practically applied in a real time market environment.

The main issue was that β demonstrated to have just a tenuous connection with real security returns in this way throwing questions on β designation as the genuine danger measure. With β addressed, and with asset pricing all in all being at somewhat of a chaos as for whether the thought of "priced risk" is truly pertinent, market practitioners hunt down a replacement risk measure that was both precise

and generally modest to evaluate. In spite of the thought of numerous different measures and models, VaR has been broadly received. Part of the reason prompting the widespread adoption of VaR was the choice of JP Morgan to make a straightforward VaR estimation model, called Risk Metrics. Risk Metrics was upheld by an openly accessible database containing the basic inputs required to appraise the model.

2.1 Review of Related Literature

Markowitz (1952) conducted spearheading work in the field of statistical market risk analysis in the mid-fifty's by presenting the Modern Portfolio Theory (MPT). In MPT, market risk is measured as the standard deviation of returns, which are expected to follow normal distribution. Market risk in MPT context thus entails both upside and drawback potential.

In recent years a concept called VaR has increased much consideration among scholastics and professionals. VaR gives an alternate way to deal with business sector risk: it is a measure of investment portfolio loss potential. VaR concentrates on the drawback risk the portfolio is presented to. Approach to risk management perceives that the risks happen just to the extent they cause financial losses. Formally VaR is characterized as the greatest expected risk over a given horizon period at a given level of confidence, (Dowd, 1998) Mandelbrot (1963) observed that volatilities of market factors are time-dependent. This phenomenon known as volatility clustering has been affirmed by numerous studies from that point forward. Therefore, in modelling market factor distributions conditional methods (time-dependent) have appeared superior to unconditional methods (time- independent) in modelling market factor distributions for VaR calculations in many empirical studies (see, e.g., JP Morgan, 1996, Goorbergh et al., 1999a). Conditional methods represent time re-liance of business sector component conveyances while unconditional methods assume that market factor distributions stay consistent after some time and are free of past realizations.

An essential milestone in the improvement of VaR models was J.P. Morgan's choice in 1994 to make its VaR framework called Risk Metrics open and accessible on the Web. The Risk Metrics framework comprises of a methodology outlining the procedures to compute VaR figures, the required market data, and software for calculations (JP Morgan, 1996). The production of the Risk Metrics urged littler organizations to embrace the Risk Metrics way to deal with VaR. In the next years the Risk Metrics framework turned into a semi-standard within the financial industry and a benchmark for measuring market risk.

VaR models have been broadly discussed in writing. As the inadequacies of the ordinary

VaR models are unquestionably comprehended, VaR-related investigation went for extra advanced methodologies with a particular deciding objective to improve the exactness and judicious power of VaR models. But new VaR approaches, for instance, Conditional Auto regressive VaR (CAVaR) have been made, there are only few studies accessible looking at a more broad extent of VaR models including both, standard and propelled VaR models.

Commercial banks may pick their administrative capital conditions for business sector risk exposure using VaR models. As of late, there are three measurable strategies (the binomial strategy, the interval forecast techniques and the distribution forecast strategy) for assessing the accuracy of VaR models that are open to Controllers. Lopez (1997) proposed another evaluation methodology considering real scoring rules for probability forecasts and the simulations results exhibited that the proposed system was clearly capable of isolating among exact and option VaR techniques.

In their article, Sarma et al (2003) performed a two relevant contextual analysis in model decision for the S&P 500 list and India's NSE-50 record, at 95% and 99%. A two-stage model determination procedure was utilized. Class of models was tried for statistical accuracy and if different models survive rejection with the tests, a second stage filtering of the surviving models using subjective loss functions. The two-stage model determination methodology wound up being sensible in picking a VaR model. The study gives affirmation about the suitability and hindrances of current data on estimation and testing for VaR.

In the contextual relationship of finance, Value at Risk is an assessment, with a sensible level of sureness, of how much a monetary pro can lose from a portfolio over a given time period. The portfolio can be that of a solitary speculator, or it can be the course of action of a whole bank. As a disadvantage risk measure, Value at Risk spotlight on low likelihood occasions that exist in the lower tail of a scattering. In setting up a hypothetical form for VaR, Jorion (1997) clarifies the critical end of period portfolio value as the worst possible end-of-period portfolio value with a pre-determined confidence level "$1-\alpha$". These most discernibly terrible qualities should not be experienced more than 1% percent of the time over the given holding period.

Recently Value at Risk is being recognized by corporate risk managers as a huge apparatus in the general risk management approach. Preliminaries interest in VaR, in any case, originated from its conceivable executions as a regulatory too. As a result of a couple of budgetary incidents including the trading of auxiliaries things, for instance, the Barrings Bank breakdown, administrative offices, for example, the Securities and Trade Commission or the BIS, in organization with a few national banks, held onto VaR as a transparent measure of downside market risk that could be useful in reporting risks related with portfolios of highly market sensitive assets such as derivatives. Since VaR focuses on disadvantage risk and is normally given in currency units, it is more consistent than other factual terms. It is frequently used for internal risk administration purposes and is further being broadcast for use in risk management purposes making by non-financial related firms.

2.2 Why Value-at-Risk

Since the distribution of JP Morgan's Risk Metrics in 1994 there has been an explosion of research in the areas of value of risk and risk management in general. While the fundamental ideas encompassing VaR are founded in the area of market risk measurement they have been extended, throughout the most recent decade, to different regions of risk management. Specifically, VaR models are presently ordinarily used to measure both credit and operational risks.

Value-at-Risk models are utilized to predict risk exposure of an investor or financial institution in the next period (day, quarter, and year). If a "bad period" happens as characterized by some statistical measure. For instance, a financial institution might wish to know its introduction to credit risk if one year from now is the worst year in 100 years (in this manner the purported 99th percentile most pessimistic scenario). With such a measure close by, the money related organization can then evaluate its capital sufficiency (in this way the measure of capital stores it has within reach to withstand such an expansive sudden misfortune). Regularly, the aggregate capital needs of a budgetary foundation have been evaluated by including required capital in various territories of risk exposure (example: market, credit, and operational risks). Notwithstanding, with the rise of different ways to deal with measuring these diverse risks, for example, such as Value at Risk, the need for a more integrative methodology turns out to be clear.

2.3 Limitations and Delimitations of Value-at-Risk

The purpose for the popularity of VaR is predominantly its conceptual simplicity as it totals every one of the risks in a portfolio into a single number suitable for use in the boardroom, answering to controllers, or divulgence in a yearly report (Linsmeier & Pearson, 1996). VaR can quantify chance over a wide range of positions (almost any asset) and risk variables (not just market risk) and it gives a fiscal and probabilistic articulation of loss amounts. In spite of critical issues, the VaR idea can be used in several ways:

- Administration can set general risk targets and from that decide the relating risk position. Expanding VaR implies expanding risk for the firm.
- VaR can be utilized to decide capital requirements. New risk based capital sufficiency system Basel II, closely resembling Basel I, endorses VaR as an essential method for measuring credit risk and consequently capital adequacy. Further, as indicated by Basel Board, banks ought to keep adequate money to have the capacity to cover business sector misfortunes more than 10 days with 99 percent likelihood for all their exchanged portfolios. This sum is to be determined by VaR, (Basle Committee on Banking Supervision, 1996).
- VaR is useful for reporting and disclosing purposes.
- VaR-based decision rules can guide investment, hedging, trading and portfolio management decisions.
- VaR information can be utilized to give compensation tenets to brokers and directors

and

- Systems based on VaR can measure other risks such as credit, liquidity and operational risks. (Dowd, 2005)

From among the critics, Taleb (1997) suggests suspension of VaR as a

- potentially dangerous malpractice as it involves principal-agent issues and is often invalid in real world settings.
- Over-reliance on VaR can lead to bigger losses.
- VaR does not portray losses beyond the particular certainty level. Danielsson & Zigrand, (2003) contend that VaR utilized for regulatory purposes might bend great risk administration practices.
- Non-coherence because of non-subadditivity of VaR is seen as the most genuine down-side of VaR as a risk measure. It must be made sub-added substance while forcing typicality confinement on return circulation, what negates the truth of financial time series.

VaR is utilized by Bank of International Settlements (BIS) for deciding capital prerequisite to cover business sector risk by typical operations. This, notwithstanding, requires the hidden risk to be appropriately assessed, else it might lead institutions to overestimate (underestimate) their market risk and therefore to keep up unnecessary high (low) capital requirements. The outcome is an inefficient allocation of financial resources. These realities were proposed by Manganelli & Engle (2004).

Among recent critics Whalen (2006) takes note that over the previous decades VaR had all the earmarks of being compelling as there was little risk to quantify and that depending on false presumption in administrative system makes VaR a standout amongst the most unsafe and generally held confusions in financial world," (Whalen, 2006, p. 2).

Chapter 3

Methodology

3.1 Introduction

This chapter explains the theory of the concept to be used, derivation and methods of analyzing the available data to satisfy the objectives of the study. It focuses on the detail and complete understanding of the performance evaluation of Value at Risk (VaR) models and its application on the Ghana Stock Exchange market.

3.2 Value at Risk (VaR)

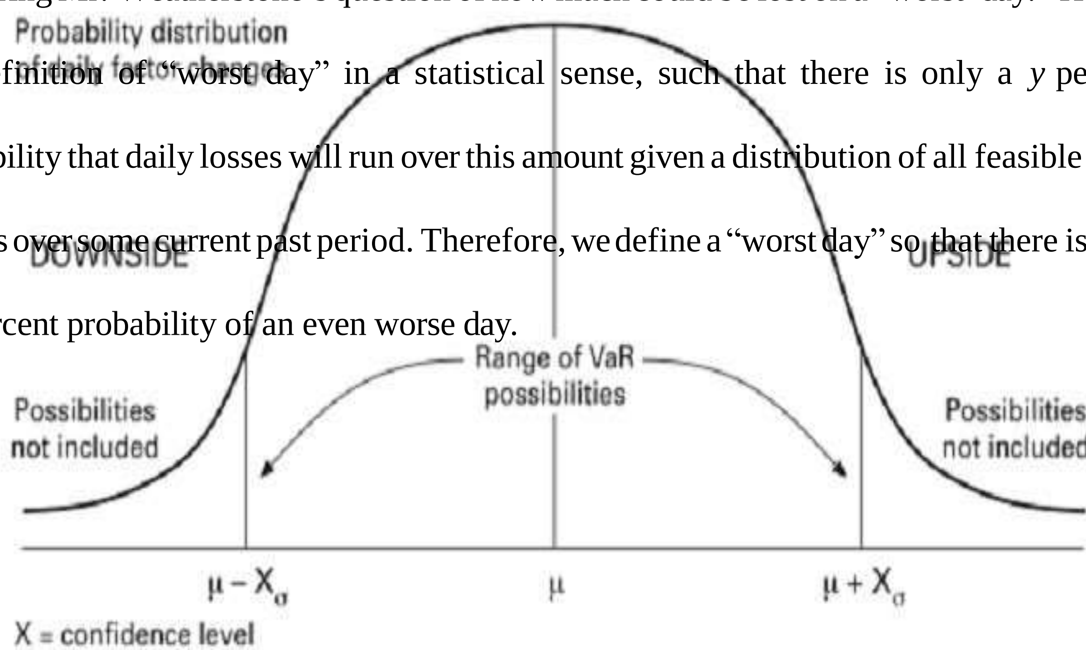
An exact computation of risk is a crucial first step for real risk management, and financial mediators, because of the nature of their business, tend to be leading developers of new risk

Dama International Journal of Researchers ISSN: 2343-6743, Scientific Journal Impact Factor: 5.968 & ISI Impact Factor: 1.018,
Dama Academia Pubisher: Vol 3, Issue 06, June, 2018, Pages 41- 83, Available @ www.damaacademia.com

measurement techniques. In the past, many of these models were internal models, developed in-house by financial organizations. As a matter of fact, the VaR tool is complementary to many other internal risk measures. Nevertheless, market forces during the late 1990s established conditions that led to the development of VaR as a main risk measurement tool for financial firms.

"how much can we lose on our trading portfolio by tomorrow's close?"

The above question was made by Dennis Weatherstone, who was at the time the Chairman of JP Morgan. There are two approaches in answering Weatherstone's question. The first is a probabilistic/statistical approach which is the center of the VaR measure and the other is the scenario approach-an event-driven, non-quantitative, subjective approach, which computes the effect on the portfolio value of a scenario or a set of scenarios that indicate what is considered adverse circumstances. VaR takes a probabilistic or statistical approach to answering Mr. Weatherstone's question of how much could be lost on a "worst day." Hence, the definition of "worst day" in a statistical sense, such that there is only a *y* percent probability that daily losses will run over this amount given a distribution of all feasible daily returns over some current past period. Therefore, we define a "worst day" so that there is only a *y* percent probability of an even worse day.



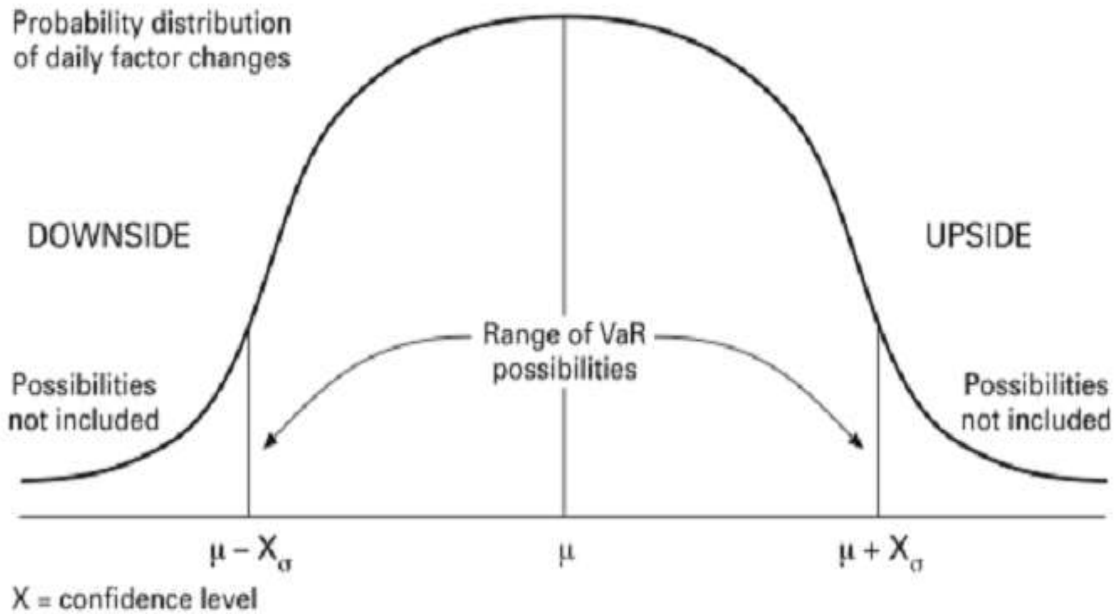


Figure 3.1: VaR and the normal distribution

3.2.1 Definition of Value-at-Risk

A value-at-risk model evaluates the market risk by determining how much the value of a portfolio could decrease over a given period of time with a given probability as a result of changes in market prices or rates. It allows managers and investors to say: *"we are X percent certain that we will not lose more than V Cedis in the next N days"* (Hull, 2002). The variable V is the Value at Risk. It is a function of two components and this components greatly affects the nature of the value-at-risk model.

- N (The time Horizon) - It is a period of time over which VaR is measured. It is traditionally measured in trading days rather than calendar days. Pragmatically, financial analysts mostly set $N = 1$, because of lack of data to estimate the behaviour of market variables over longer period of time.
- Y (The Confidence level) - Frequently used confidence levels are 99% and 95%. For instance, a 500 Cedis, one day, 95% confidence level VaR value for a stock means that during the next day we are 95% certain that the value of our asset in this specific stock will not decrease by more than 500 Cedis. The VaR will decrease if a lower confidence level say 99% or 95% is chosen. Different confidence levels will suit different organizations and purposes and will be chosen according to financial analysts relation to risk. The more risk averse the firm is the higher the confidence level will be selected.

To provide an overview of the measure of VaR, a simple example is given. Assume that the unit share price of Tullow oil on the Ghana Stock Exchange market today is 20 Cedis and the daily standard deviation (σ) is 4 Cedis. Investors that purchase larger shares might want to know how much, given a certain confidence level, they can possibly lose when purchasing the share today compared to tomorrow. Suppose, the chosen confidence interval is 99%, this means that a day out of hundred, the loss will be greater than the calculated VaR. This is true when the share price is normally distributed around the mean price change.

Value due to a decrease in the share price $= 20 - 2.33 \text{ x } \sigma = 10.68$ Value due to an increase in the share price $= 20 + 2.33 \text{ x } \sigma = 29{:}\ 32$

This can be explained as, with a 99% probability, the loss will not be greater than $20 - 10.68 = 9.32$ Ghana Cedis which is the VaR for a confidence level of 99%.

3.2.2 Assumptions behind Value-at-Risk

As often as possible, some measurable presumptions are made keeping in mind the end goal to compute the VaR. The stationarity prerequisite. That is, a 1% change in returns is similarly prone to happen anytime. Stationarity is a customary presumption in money related financial aspects, since it disentangles calculations significantly. A related presumption is the random walk assumption of inter-temporal unpredictability. That is, everyday varieties in returns are autonomous; say, a lessening in the Ghana Stock file on one day of *y*% has no prescient force concerning returns on the Ghana Stock record on the following day. Additionally, the random walk assumption can be depicted as the presumption of a normal rate of return equivalent to zero, as in the value portfolio sample. Henceforth, if the normal day by day return is zero, then the ideal speculation assessment of tomorrow's value level is today's level.

A basic supposition is the non-negativity requirement, which obviously expresses that money related resources with limited obligation can't accomplish negative qualities. All things considered, subsidiaries (examples: forwards, futures, and swaps) can repudiate this presumption. The time consistency necessity declares that all unit period suppositions hold over the multi-period time horizon.

Another most noteworthy suspicion is the distributional assumption. In the basic equity portfolio illustration, it can be expected that every day return varieties in the Ghana Stock file take after a typical dissemination. The supposition has the upside of making the VaR estimations much less demanding. In any case it has a few downsides. The value changes don't generally suit the typical appropriation bend and when more perceptions are found in the tails, ordinary based VaR will downplay the misfortunes that can happen.

3.2.2 Steps in calculating Value-at-Risk

There are three steps in VaR calculations. The first step is the holding period, the time period over which the losses may occur. This period is mostly a day, however it can be more or less conditional on a particular situation. Investors who actively trade their portfolios has the tendency to use a 1-day holding period, whereas longer holding periods are more pragmatic for nonfinancial firms and institutional investors. The longer the holding period, the larger the VaR.

The next step is the probability of losses more than VaR, p, needs to be stated, with the most ordinary probability level being 1%. Literature provides little direction about the choice of p; it is mainly decided by how the user of the risk management system desires to explain the VaR number. VaR levels of 99% to 95% are ordinary in practice, however less extreme higher numbers are often used in risk control on the trading floor and most extreme lower numbers may be used for applications like survival analysis, economic capital, or long-run risk analysis.

The last step is to determine the probability distribution of the profit and loss of the portfolio. This is the most problematic and significance aspect of risk modeling. The normal practice is to evaluate the distribution by using historical observations and a statistical model. Calculate the VaR estimate - this is accomplished by observing the loss amount related with that area under the normal curve at the critical confidence interval value that is statistically related with the probability chosen for the VaR estimate in step 2.

3.2.4 Interpreting and Analyzing Value-at-Risk

In deciphering and looking at VaR numbers, it is basic to give careful consideration to the likelihood and holding period since, without them, VaR numbers are valueless. Case in point, a portfolio containing the same resource could deliver two divergent VaR estimates if risk managers choose different values of the probability of losses more than VaR and holding periods. Plainly, a loss permitted with a likelihood of just 3% rises above a loss permitted with a likelihood of 7%. With regards to the VaR of an organization's arrangement of positions is a related measure of the risk of budgetary anguish over a brief period depend on the liquidity of portfolio positions and the risk of amazing money outpourings. Antagonistic liquidity conditions lead to high exchange costs, for example, wide spreads and large margin calls. VaR is unrealistic to catch these effects.

In danger enhancement, VaR is an imperative stride forward as for customary measures in view of susceptibilities to market variables. VaR is a complete thought and can be actualized to most money related instruments. It encapsulates in a single number all the risks of a portfolio fusing loan cost hazard, remote trade hazard. It additionally speeds up examinations between disparate resource classes. The VaR measure consolidates quantile (loss) and likelihood.

3.2.5 Measuring Returns

From the definition of VaR, the VaR number is the portfolio return in the worst case, hence the definition of portfolio return is first introduced. The return on Portfolio, ΔP is the difference between portfolio values, that is, $\Delta P = P_{t+1} - P_t$, where the portfolio values at time t and $t+1$ are P_t and P_{t+1} respectively. The portfolio returns can be described by the rate of return. Nonetheless, there are two kinds of rates, namely the arithmetic and geometric. Geometric rate of reyuum R_g is the logarithm of theprice ratio, mathematically, $R_g = \ln \frac{P_t}{P_{t-1}}$ whilst arithmetic rate of return R_α, is portfolio return divides the original value , mathematically $R_\alpha = \frac{P_t - P_{t-1}}{P_{t-1}}$

By substitution, it can be noted that $R_g = \frac{P_t - P_{t-1}}{P_{t-1}} = \ln(1 + R_\alpha)$. Suppose the time horizon is short, the daily arithmetic rate of return is around zero, then by Taylor expansion,

$$R_g = R_\alpha - \frac{R_\alpha^2}{2} + \frac{R_\alpha^3}{3} - \frac{R_\alpha^4}{4} + \ldots$$

Hence $R_\alpha \cong R_g$, implying that arithmetic rate and geometric rate are the same and we can use R to denote both. Let $R_{t,n}$ be the rate of return during the last n days, by geometric rate of returns:

$$R_{t,n} = \ln(\frac{P_t}{P_{t-1}}) = \ln(\frac{P_{t-1}}{P_{t-2}}) + \ldots + \ln(\frac{P_{t-n+1}}{P_{t-n}}) = R_t + R_{t-1} + \ldots + R_{t-n+1}$$

The rate of return during the last n days is the sum of n proceeding rates. The portfolio return of two consecutive days is $R_{t,2} = R_t + R_{t-1}$.

3.2.6 Deriving Value-at-Risk

The loss on a trading portfolio such that there is a probability p of losses equaling or exceeding *VaR* in a given trading period and a (1 - p) probability of losses being lower than the *VaR*.

Mostly written as VaR (p) or *VaR* $100 \times P\%$ making the reliance on probability clear for instance, *VaR* (0.01) or *VaR* 1%. Probability levels mostly used in calculating *VaR* is 99% and 99.5%, however, percentage values that are lower and higher than these are mostly used in its application.

VaR is a quantile on the distribution of Profit and Loss $(P = L)$. Let the random variable R denote the $(P = L)$ on an investment portfolio, with a specific realization say r. If one unit of an asset is held, ($P = L$) can be written as:

$$R = P_t - P_{t-1} \tag{3.1}$$

Comprehensively, if the portfolio value is ψ and the returns is Y, then:

$$R = \psi Y \tag{3.2}$$

Let the density of P/L be denoted by *fr* (.), then VaR is given by:

$$\Pr\left[R \leq -VaR(p)\right] = p \tag{3.3}$$

$$p = \int_{-\inf}^{-VaR(p)} f_r(y)dy \tag{3.4}$$

From equations (3.1) - (3.4), *VaR* can be derived from simple returns,

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Assuming the return is zero and volatility is indicated by σ, then from the definition of *VaR* in (3.3) and (3.4), *VaR* can be obtained from:

$$p = P_r(P_t - P_{t-1} \leq -VaR(p))$$

$$p = P_r(P_{t-1} - R_t \leq -VaR(p))$$

$$p = P_r(\frac{R_t}{\sigma} \leq -\frac{VaR(p)}{P_{t-1\sigma}})$$

Let the distribution of standardized returns $\frac{R_t}{\sigma}$ be QR(.) and the distribution by $Q^{-1}(p)$.

Hence the VaR for holding a unit of the asset is:

Dama International Journal of Researchers ISSN: 2343-6743, Scientific Journal Impact Factor: 5.968 & ISI Impact Factor: 1.018,
Dama Academia Pubisher: Vol 3, Issue 06, June, 2018, Pages 41- 83, Available @ www.damaacademia.com

$VaR(p) = -\sigma Q_R^{-1}(p) P_{t-1}$

The significance level can be denoted by $y(p) = Q_R^{-1}(p)$ the VaR equation can be written as:

$$VaR(p) = -\sigma y(p) P_{t-1}$$

However if continuously compounded returns are used:

$Y_t = \log(1 + R_t) = \log(\frac{P_t}{P_{t-1}})$

$Y_t = \log P_t - \log P_{t-1}$

This implies that,

$p = P_r(P_t - P_{t-1} \le -VaR(p))$

$p = P_r(P_{t-1}(e^{Y_{t-1}}) \le -VaR(p))$

$p = P_r(\frac{R_t}{\sigma} \le \log(-\frac{VaR(p)}{P_{t-1}} + 1)\frac{1}{\sigma})$

Since $-\frac{VaR(p)}{P_{t-1}} \le 1$ we can denote the distribution of standardized returns $\frac{Y_t}{\sigma}$ by $Qy(.)$

the inverse distribution by $y(p) = Q_y^{-1}(p)$, we have

$$VaR(p) = (-\exp(Q_y^{-1}(p)\sigma) - 1)P_{t-1}$$

and for small $Q_y^{-1}(p)\sigma$, the *VaR* for holding one unit of the asset is given by:

$VaR(p) \approx -\sigma y(p) P_{t-1}$

So, the *VaR* for continuously compounded returns is approximately the same as the *VaR* using simple returns.

3.3 Coherence

The properties a risk measure should have in order to be considered a functional risk measure was studied by Artzner et al. (1999); they determined four axioms that risk measures ideally should comply with. If a risk measure satisfies these four axioms it is called coherent. Let τ denote a risk measure. In this work, our risk measure is the VaR.

Definition 3.6.1

Consider two real-valued random variables: A and B. A function $\tau(.)$: $A, B \rightarrow \Re$ is called a coherent risk measure if it satisfies for A, B and constant k. V is the VaR.

1. Subadditivity

   $A, B, A+B \in V \Rightarrow \tau(A+B) \leq \tau(A) + \tau(B)$

   The risk to the portfolios of A and B cannot be worse than the sum of the two individual risks-an illustration of the diversification principle.

2. Translation Invariance

   $A \in V, k \in \Re \Rightarrow \tau(A+k) = \tau(A) - k$

   Adding k to the portfolio is like adding cash, which acts as insurance, so the risk of $A+k$ is less than the risk of A by the amount of cash, k.

3. Positive Homogeneity

   $A \in V, k > 0 \Rightarrow \tau(kA) = k\tau(A)$ for $k > 0$

For instance, if the portfolio value doubles ($k = 2$) then the risk doubles.

4. Monotonicity

$A, B \in V, A \leq B \Rightarrow \tau(A) \geq \tau(B)$

If portfolio A never transcends the values of portfolio B (that is, it is always more negative, consequently, its losses will be equal or larger), the risk of B should never surpass the risk of A.

However the axiom of positive homogeneity is pragmatically violated. For instance, suppose the maximum risk a portfolio worth five hundred Ghana Cedis can hold is thirty Cedis. Then this implies from a axiom 3 that, whenever the portfolio is doubled, the risk should also be doubled. But this is always not the case because as relative shareholdings increase and/or the liquidity of a stock decreases, risk may increase more rapidly than the portfolio size. In such a situation positive homogeneity is violated and:

$$\tau(kA) > k\tau(A) \tag{3.5}$$

Among these four axioms, the most important is the sub-additivity. A portfolio of assets is measured as less risky than the sum of the risks of distinct assets if this axiom holds. If VaR violate this axiom, it can erroneously be concluded that diversification results in an increase in risk. VaR is sub-additive in the special case of normally distributed returns. Daníelsson et al. (2010a) studied the sub-additivity of VaR in details and found out that VaR is actually sub-additive conditioned that the tail index exceeds 2- when the second moment, or variance, is defined under a condition of multivariate regular variation.

3.4 VaR Approaches

Historical simulation (HS) method

Historical simulation can also be used in estimating the Value at Risk. Historical Simulation is more pliable than the Parametric method and avoids some of the pitfalls of the parametric method. This method has the benefit of simply handling options in the portfolio (Best, 1998). It also has the benefit of extensively accepted by trading communities and management mostly because of its clarity. The historical simulation method calculates potential losses using real historical data of the returns in the risk factors and hence captures the non-normal distribution of risk factor returns. Because the risk factor returns used for revaluing the portfolio are real past movements, the correlation in the estimation are also actual historical correlations. As Daníelsson (2011) clearly stated, the main concept of this methodology is to predict future losses based on the historical performance. Historical simulation (HS) is a simple method for forecasting risk and relies on the assumption that history repeats itself, where one of the observed historical returns is anticipated to be the next period return. Each historical observation carries the same weight in HS forecasting. This can be a disadvantage, specifically when there is a structural break in volatility. Nevertheless, in the absence of structural breaks, HS tends to function better than alternative methods. It is less sensitive to the odd outlier and does not absorb estimation error in the same way as parametric methods. The importance of HS become especially clear when working with portfolios because it directly captures nonlinear dependence in a way that other methods cannot.

Values of the market components for a specific past period are fetched and changes in these values over the time horizon are observed for use in the calculation. For example, if a 1-day VaR is needed using the past 50 trading days, each of the market factors will have a vector of observed changes that will be made up of the 49 changes in value of the market factor. A vector of different values is generated for each of the market factors by adding the contemporary value of the market factor to each of the values in the vector of observed changes.

The portfolio value is constructed using the present and alternative values for the market factors. The variations in portfolio value between the recent value and the alternative values are then evaluated. The last step is to categorize the changes in portfolio value from the smallest value to highest value and ascertain VaR based on the required confidence interval. For a 1-day, 95% confidence level VaR using the past 100 trading days, the VaR would be the 95th most unfavourable change in portfolio value.

The risk is calculated with price changes:

1. Absolute change in price,
2. Logarithmic change in price,
3. Relative change in price, but should the change be relative to the initial price, then

it is called return or rate of return.

1-day Period

The price in time t can be denoted as P_t (which represents one trading day). The relative rate of return (R_t), between t and $t-1$ can be calculated as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

The logarithmic rate of return (Rl_t) correspond to

$$(Rl_t) = \log \frac{P_t}{P_{t-1}} = \log(1 + R_t)$$

The absolute rate of return (Ra_t) for the same time period is

$$Ra_t = P_t - P_{t-1}$$

K- days Period

Return of the k-days period of time is defined as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-k}}$$

The main assumptions of HS are:

- Selected sample period could describe the properties of assets very well,
- There is a probability of reiterating the past in the future, that is, the recreation of the patterns appeared in the volatilities and correlations of the returns in historical sample, in the future. However, the past could be a good basis of the future forecast.

The process used to estimate the VaR of a given portfolio using historical simulation is as below:

1. A portfolio of M assets denoted by a vector of weights is defined;

$$\bar{\omega} = \begin{pmatrix} \omega_0, 1 \\ \omega_0, 2 \\ \omega_0, 3 \\ \vdots \\ \omega_0, M \end{pmatrix} \tag{3.6}$$

2. For each asset price or risk factor involved in the problem, obtain a series of returns for a given time period (for example, 200 days). When log-returns are used, they are calculated as below:

$$r_{k,t} = \log \frac{pk,t}{pk,t-1} \tag{3.7}$$

where $r_{k,t}$ and pk,t are respectively the return and price of the asset *k* at time *t*.

3. Consider each of the days in the time series of returns as a scenario for possible Changes in the next day. As there are M assets, each day t of historical data will form a scenario defined by:

$$\bar{r}_t = \begin{pmatrix} r_1, t \\ r_2, t \\ r_3, t \\ \vdots \\ \omega M, t \end{pmatrix} \tag{3.8}$$

It is important to notice that from this point on the scenarios $\bar{r}_t$ are no longer seen as time series, but just as a set of different possible realizations of the random vectors $\bar{r}_t$, obtained from historical data.

4. Apply each of the scenarios to the composition of the portfolio today, that is, do not apply the price changes in cascade to the portfolio. Indicating that the outcome of the application of scenario t to the portfolio is:

$$\bar{\omega} = \begin{pmatrix} \omega_0, 1 \\ \omega_0, 2 \\ \omega_0, 3 \\ : \\ \omega_0, M \end{pmatrix} = \begin{pmatrix} \omega_0, 1.e^{r1,t} \\ \omega_0, 2e^{r2,t} \\ \omega_0, 3e^{r3,t} \\ : \\ \omega_0, Me^{rM,t} \end{pmatrix} \tag{3.9}$$

Note that despite the fact that the notation $w_{t,k}$ is used to represent weights in Pth portfolio, they will not be normalized in this procedure, in such a way that $\sum_{k=1}^{N} w_{t,k}$ for $k \neq 0$ may be different than one.

5. The log-returns of the portfolio for each of the scenarios are estimated as:

$$R_t = \log(\sum_{k=1}^{N} w_{t,k}) \tag{3.10}$$

remembering that $\sum_{k=1}^{N} w_{t,k} = 1$

6. Categorize the portfolio returns (R_t) for the various scenarios into percentiles.
7. The VaR will be the return that correlate with the preferred confidence level. For instance, if there are 200 days and a confidence level of 99% preferred, the VaR will be the second worst return of the portfolio.

Monte Carlo Simulation method

Monte Carlo simulation is more pliable. Unlike historical simulation, Monte Carlo simulation permits the risk manager to use real historical distributions for risk factor returns as opposed to having to assume normal returns. Monte Carlo simulation is an extensive method of stochastic modeling processes-processes entailing human selection for which we have insufficient information. It imitates such a procedure by way of random numbers obtained from probability distributions which are presumed to correctly describe the un- known constituents of the process being modeled. Monte Carlo simulation is largely used in physics and engineering as well as in finance.

Stanislaw Ulam created the Monte Carlo approach in 1946 (Eckhardt, 1987) and includes some method of statistical sampling used to estimate solutions to quantitative problems. In the procedure, the arbitrary procedure under analysis is imitated time after time, where in each simulation will be generated a scenario of conceivable parameters of the portfolio at the target horizon. By creating a substantial number of plans, ultimately the distribution acquired through simulation will converge towards the true distribution. A good illustration of this method can be obtained, for example, in Holton (2003, chapter 5).

Crouhy et al. (2001) stated that this approach is beneficial in that: it allows the performance of sensitivity analyses and stress testing; the method can be used to model any complex portfolio; and that any distribution of the risk factors may be used. He however stated that outliers are not incorporated into the distribution; it is very computer intensive. In addition to the strength of Monte Carlo simulation is that no assumptions about normality of returns have to be made. The method is also capable of covering nonlinear instruments, such as options, Damodaran (2007). To add more to the benefits of this approach of VaR, Jorion (2001) reminds that Monte Carlo simulation initiates the whole distribution and consequently it can be used, for example, to estimate losses in excess of VaR. A possible weakness is also model risk, which arises due to wrong assumptions about the pricing models and underlying stochastic processes, a possible weak. If these are not properly stated, VaR calculations will be misrepresented, Jorion (2001).

Furthermore, Dowd (1998) points out that complex techniques related to this approach necessitate specific skills. Senior management may therefore have difficult time acquainting themselves of how VaR values are calculated when Monte Carlo is used.

3.5 Comparison of Methods

VaR methods vary in their propensity to capture risks of options, ease of execution, ease of interpretation to directors and managers, pliability in analyzing the effect of variations in the assumptions, and reliability of the results (Linsmeier and Pearson, 1996).

As for the accuracy of the result, the best method seems to be the Monte Carlo method. The advantage exibility is particularly large. However, its use may be time consuming and it requires some knowledge and experience of the creators and users. Both Monte Carlo simulation and historical simulation methods rely on simulations and they suffer when using a lower number of scenarios by bad convergence to the actual sample quantile. While

Monte Carlo method is generating larger number of scenarios, and the limits are given by the computational resources available, the historical simulation method exhibits a more serious problem a long time series are often not available and VaR cannot be estimated, especially at higher levels of probability.

The optimal choice will be decided by which dimension the risk manager finds most significant and appropriate. If VaR is being calculated for a risk source that is stable and in the presence of real historical data, historical simulations provide good estimate. In the most comprehensive case of computing VaR for nonlinear portfolios over long time periods, where the historical data is volatile and non-stationary and the normality assumption is uncertain,

Monte Carlo simulations do best (www.stern.nyu.edu/ adamodar/pdfiles/papers/VAR.pdf, 2010).

3.6 A General Solution to the Basic VaR Problem

Given that a portfolio consist of Assets $1,2,3,\ldots,N.D_i$ Cedis are invested in Asset i, the total value of the portfolio is then then

$$D_1 + D_2 + D_3 + \ldots + D_N = \sum_{i=1}^{N} D_i = D \text{ Cedis.}$$

Assumption is made that the one-day Asset i return is normally distributed with variance σ_i^2 and expected value $E(r_i)$. The covariance between the 1-day returns of Assets i and j is denoted by σ_{ij}. We want to determine the 1-day VaR at a confidence level of 5%

To solve this problem above, calculate the variance and expected return of the total portfolio. To do this, the weighting for each asset is calculated. The proportion of the portfolio expected return located to Asset ***i*** is $\psi_i = \frac{D_i}{D_1 + D_2 + D_3 + ... + D_N}$

These are called asset weighting factors. Let $K = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ ... \\ \psi_N \end{pmatrix}$ and $U = \begin{pmatrix} E|r_1| \\ E|r_2| \\ E|r_3| \\ ... \\ E|r_M| \end{pmatrix}$

A linear combination of random variables are created, where the random variables are the expected 1-day returns for each asset, and the coefficients are the asset weighing factors.

From the property of expectations,

$$E\left|\sum_{i=1}^{n} \alpha_i X_i\right| = \sum_{i=1}^{n} \alpha_i E|X_i|$$

and using the matrix method for finding this expectation: $E|r_{portfolio}| = E\left|\sum_{i=1}^{N} \alpha_i r_i\right| =$

$$K^T U = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ ... \\ \alpha_N \end{pmatrix}^T \begin{pmatrix} E|r_1| \\ E|r_2| \\ E|r_3| \\ ... \\ E|r_M| \end{pmatrix}$$

$$E|r_{portfolio}| = \sum_{j=1}^{M} \psi_j E|r_i| = \mu_p$$

Subsequently, the variance of the total portfolio is calculated. The variance of the linear combination of random variables is given by:

$$Var(\sum_{i=1}^{n}\alpha_i X_i) = \sum_{i=1}^{n}\alpha_i^2 Var(X_i) + 2\sum_{i\text{p} j}\sum^{N}\psi_j\psi_i Cov(r_i, r_j)$$

The above equation is modified to the conditions of our stated problem:

$$\alpha_P^2 = Var(\sum_{i=1}^{N}\alpha_i r_i) = \sum_{i=1}^{N} alpha_i^2 Var(r_j) + 2\sum_{i\text{p} j}\sum^{N}\psi_j\psi_i Cov(r_i, r_j)$$

$$\alpha_p^2 = \sum_{i=1}^{N}\alpha_i^2\alpha_j^2 + 2\sum_{i\text{p} j}\sum^{N}\alpha_j\alpha_i\alpha_{ij}$$

By computing α_p^2,

$$K = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ ... \\ \alpha_N \end{pmatrix}^T \text{ and } \sum = \begin{pmatrix} \alpha_1^2 & \alpha_{12} & \alpha_{13} & ... & \alpha_{1N} \\ \alpha_{21} & \alpha_2^2 & \alpha_{23} & ... & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_3^2 & ... & \alpha_{3M} \\ . & . & . & . & . \\ \alpha_{N1} & \alpha_{N2} & \alpha_{N3} & ... & \alpha_N^2 \end{pmatrix}$$

From the definition that:

$$Var(\sum_{j=1}^{n}\alpha_i X_i) = K^T \sum K$$

$$\Rightarrow \alpha_P^2 = Var(\sum_{j=1}^{n}\alpha_i r_i) = K^T {}_{\sum K}$$

$$\alpha_P^2 = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \dots \\ \alpha_N \end{pmatrix}^T \begin{pmatrix} \alpha_1^2 & \alpha_{12} & \alpha_{13} & \dots & \alpha_{1N} \\ \alpha_{21} & \alpha_2^2 & \alpha_{23} & \dots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_3^2 & \dots & \alpha_{3M} \\ . & . & . & . & . \\ \alpha_{N1} & \alpha_{N2} & \alpha_{N3} & \dots & \alpha_N^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \dots \\ \alpha_N \end{pmatrix}$$

The VaR can now be calculated since the expected value and variance for the overall portfolio return has been evaluated. Assume that the portfolio return is normally distributed with mean μ_P and variance α_P^2 both of which has already been calculated for. Since a 5% confidence level is needed, the return is solved such that a return worse than this return occurs only 5% of the time.

Mathematically, we are solving for r*. Assume r* is found. Normally r* is minute, non-positive decimal. 100r*% is a percentage and can be thought of as the one-day percent loss such that, in normal market conditions, the portfolio loses more than 100 r*% only 5% of the time. Therefore, the one-day Value-at-Risk at a 5% confidence level is

$D\,|r*|$.

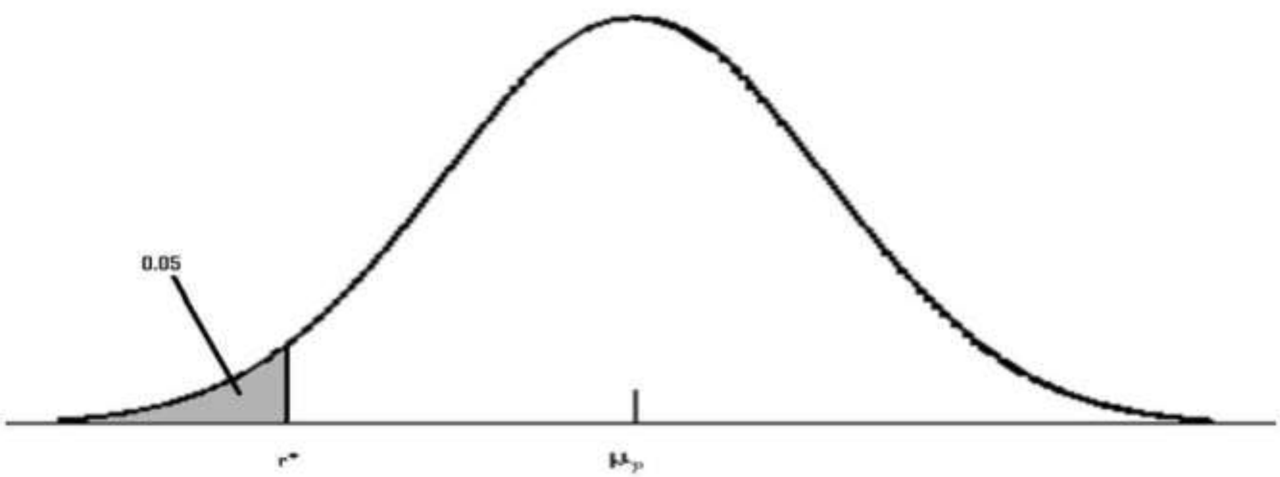


Figure 3.2: Line plot of returns of stock 1

In the rare occurrence that $r* > 0$, the VaR is not very beneficial. However, r* was evaluated such that the portfolio performs worse than r* only 5% of the time. But $r* > 0$, hence it can be stated that only 5% of the time will the portfolio earn us a positive return between 0 and r* or lose money. Therefore, if $r* > 0$ is obtained, it is an ineffective metric.

A new VaR analysis should then be estimated with a lower confidence level until we obtain an $r* < 0$.

3.3 Skewness

Skewness is a measure of the asymmetry of the data around the sample mean. If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or any perfectly symmetric distribution) is zero. This means normally distributed data is assumed to be symmetrically distributed around its mean. The skewness of a distribution is defined as

$$s = \frac{E(x-\mu)^3}{\alpha^3} \tag{3.11}$$

Where μ is the mean of x, σ is the standard deviation of x, and E(t) represents the expected value of the quantity t. Skewness computes a sample version of this historical value.

Therefore a dataset with either a positive or negative skew deviates from the normal distribution assumptions. This can cause parametric approaches of VaR to be less effective if assets returns are heavily skewed, since these approaches assume that the returns are normally distributed.

3.3 Kurtosis

Kurtosis is a measure of how outlier-prone a distribution is. The kurtosis measures the peakedness of a data sample and describes how concentrated the returns are around their mean. A high value of kurtosis means more of the data's variance comes from extreme deviations.

The kurtosis of a distribution is defined as

$$k = \frac{E(x-\mu)^4}{\alpha^4} \qquad (3.12)$$

Where μ is the mean of x, σ is the standard deviation of x, and E(t) represents the expected value of the quantity t.

Dama International Journal of Researchers ISSN: 2343-6743, Scientific Journal Impact Factor: 5.968 & ISI Impact Factor: 1.018, Dama Academia Pubisher: Vol 3, Issue 06, June, 2018, Pages 41- 83, Available @ www.damaacademia.com

Chapter 4

Data Collection and Analysis

4.1 The Underlying Assets

The fundamental stocks, on which our VaR calculations are based, are chosen because of their basic differences which could indicate fundamentally differing return distributions. All six (6) stocks show diverse properties making them compelling for comparison according to the purpose of the study.

Anytime an asset is held, the asset face a risk of gaining or losing value in relation to its closing price. The scope of risk management is all about evaluating and extenuating risk. A successful risk management is according to Saunders & Cornett (2007) central to a financial institutions performance. This is also true for any company exposed to risk. as discussed in the problem discussion VaR is a useful measure for all of our assets.

4.2 Simulation and Presentation of the Results

We randomly selected 6 companies data from the Ghana's Stock Exchange (GSE) and used it to demonstrate the evaluation of Value at Risk on the Ghana Stock Exchange. With diverse methods and models, the difficulty that VaR users encounter is the choice of picking the appropriate one that will be compatible to their purpose. The methods should make approximations that suits the future distribution of returns. If VaR is overestimated, then users ends up with an overestimation of the risk resulting in exorbitant holding of amounts of cash to cover losses. In this thesis, VaR will be calculated for six (6) different underlying assets and the results would be compared. The assets that will be used for the calculations have been labeled Stock 1, Stock 2, Stock 3, Stock 4, Stock 5, and Stock 6. The VaR approaches that will be used in the calculation are historical simulation, Monte Carlo approach. When using the parametric approaches, it becomes suspicious that the returns that the calculations are based on are most likely not to be perfectly normally distributed. The performance of these parametric methods will consequently partly be determined by how appropriate the normal distribution assumption fits the actual distribution of the re- turns.

The tool that we have used for our calculation is Matrix Laboratory ($MATLAB$). $MATLAB$ is a very versatile tool that can be helpful if one know how to use it right. It has several toolbox including financial toolbox hence its versatility and several functions that are automated functions for calculating a lot of operations.

4.2.1 The Data

The data used in the study are time series that exhibits the historical price changes for the chosen stock's. Historical data was collected for period of 1835 trading days (from 2007 to 2014) in order to get 1834 daily returns. It is important to have complete historical data of stock prices in order to calculate VaR. These historical prices consist of the daily closing prices (the last value for any specific day) of the various stock's. Portfolio returns are calculated as weighed sum of individual returns. The main benchmark for choosing stocks into portfolio was their liquidity. The reason being that it is important to have complete historical data of stock prices in order to calculate VaR.

Some characteristic key figures about the data sets are calculated in order to better be able to compare them. The key figures that we have calculated are skewness, kurtosis, standard deviation, volatility, the minimum and maximum values. The correlation coefficient and the covariance matrix was also calculated as the input data. These historical data are taken from companies that have different economic characteristic.

Table 4.1: PER YEAR RETURNS OF SIX (6) STOCKS ON THE GSE MARKET

	STOCK 1	STOCK 2	STOCK 3	STOCK 4	STOCK 5	STOCK 6
2014	-0.0004303	-7.11E-05	-0.0006818	0.00040769	-0.000156	0.00116292
2013	-0.0014531	-0.0016411	-0.0008047	-0.0010912	-0.0014658	-0.0015084
2012	-0.0004292	-0.0005435	-0.0003245	-0.0007192	-0.0002256	-0.0009574
2011	-0.0007452	6.17E-05	0.00071292	0.00262885	0.00065822	3.41E-05
2010	-0.0007525	-0.0006896	4.27E-12	-0.0011956	-0.0023954	-0.0002683

2009	0.00133932	0.00200471	0.00200471	-0.0003827	0.00072337	0.00071721
2008	-0.0012442	-0.000533	0.00182533	-0.0011037	-0.000175	-0.0008479
2007	0.00036378	-0.0016855	-0.0003406	-0.0006783	-0.0012977	-0.0008711

Table 4.2: STATISTICAL CHARACTERISTICS OF STOCKS RETURNS

	STOCK 1	STOCK 2	STOCK 3	STOCK 4	STOCK 5	STOCK 6
EXPECTED RETURN	−0.0004189	−0.0003872	0.00029891	−0.0002668	−0.0005417	−0.0003174
ANNUAL VOLATILITY	11.5469597	19.1699283	17.2571318	23.3475277	16.8643871	11.8140655
STANDARD DEVIATION	0.00090021	0.0011599	0.00110052	0.00128007	0.00108792	0.00091057
SKEWNESS	0.87402115	0.94068273	0.66145097	1.63635186	−0.4387691	0.42957594
KURTOSIS	2.87487752	3.43707005	1.85428093	4.39507528	2.04507954	1.97500325

4.3 Stocks

Stock 1

With an annualized volatility of 11.54696%, the prices of Stock 1 is the less volatile among all the six (6) stocks. Associated with an annualized returns of -0.004189, the standard deviation on the returns is 0.00090. In figure 4.2, a normal probability distribution curve have been fitted against a histogram of the returns. The kurtosis is 2.87488, which makes it lower with fat tails than the normal distribution. The distribution of returns is less outlier prone since the value of the kurtosis is less than 3. However, the graph in 4.2 is positively skewed with skewness of 0.87402. This indicates that the returns are skewed to the left, causing parametric approaches of VaR to be less effective, since these approaches assume that the returns are normally distributed.

Stock 2

The annualized volatility of stock 2 was 19.16993% with an associated returns of -0.00039. However, it recorded the lowest deviation of returns from the average returns of all the six stocks. Clearly, the skewness of figure 4.4 is highly positively skewed, 0.94068. With a kurtosis of 3.43707, the distribution of returns on the 4.4 is outlier prone. There are large outliers in that they are not clearly perceptible in the graph which lie outside the area of the normal distribution curve.

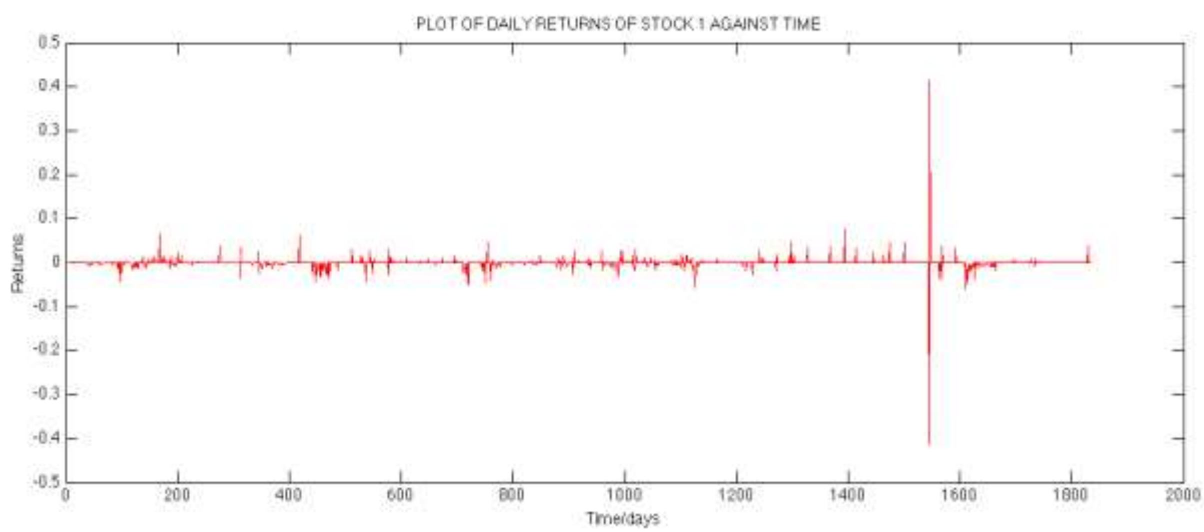


Figure 4.1: Line plot of returns of stock 1

Stock 3

Stock 3 recorded the highest possible annualized returns of 0.000299 and the associated volatility was 17.25714%. Characterized with the lowest kurtosis of 1.85428, indicating that the plot in 4.6 is less outlier-prone. The distribution is positively skewed, implying that the data are spread out more to the right than to the left.

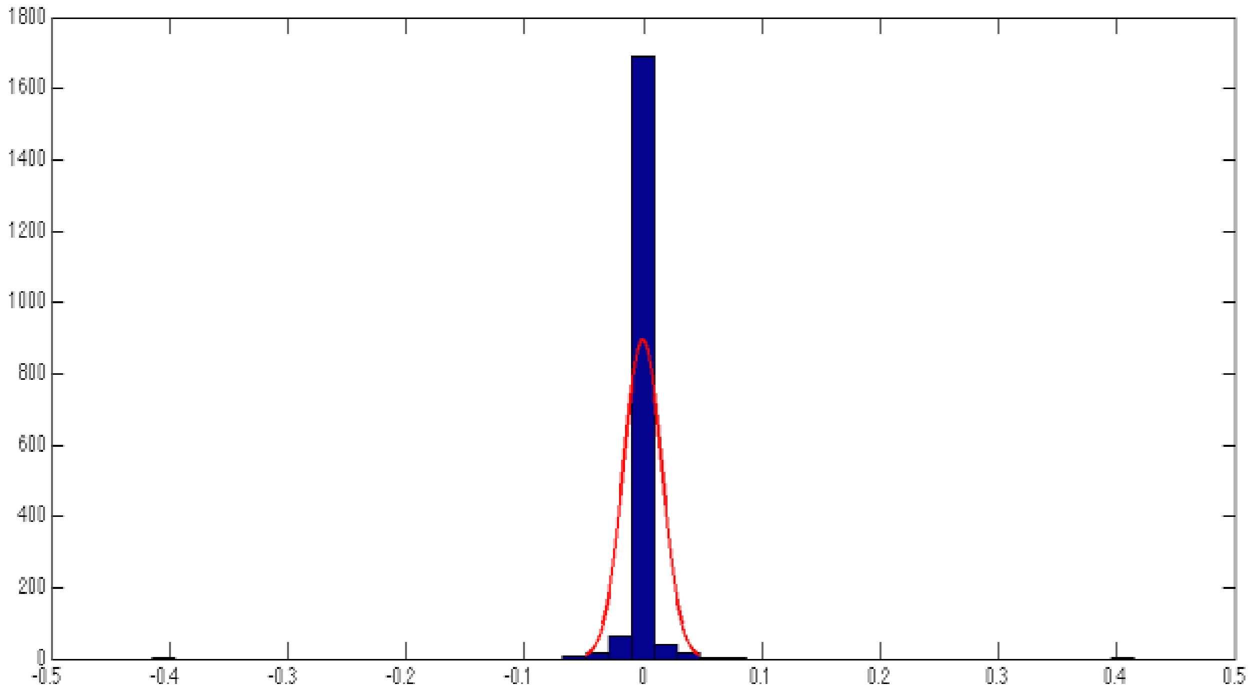


Figure 4.2: Histogram showing the daily returns of Stock 1 combined with a normal distribution curve

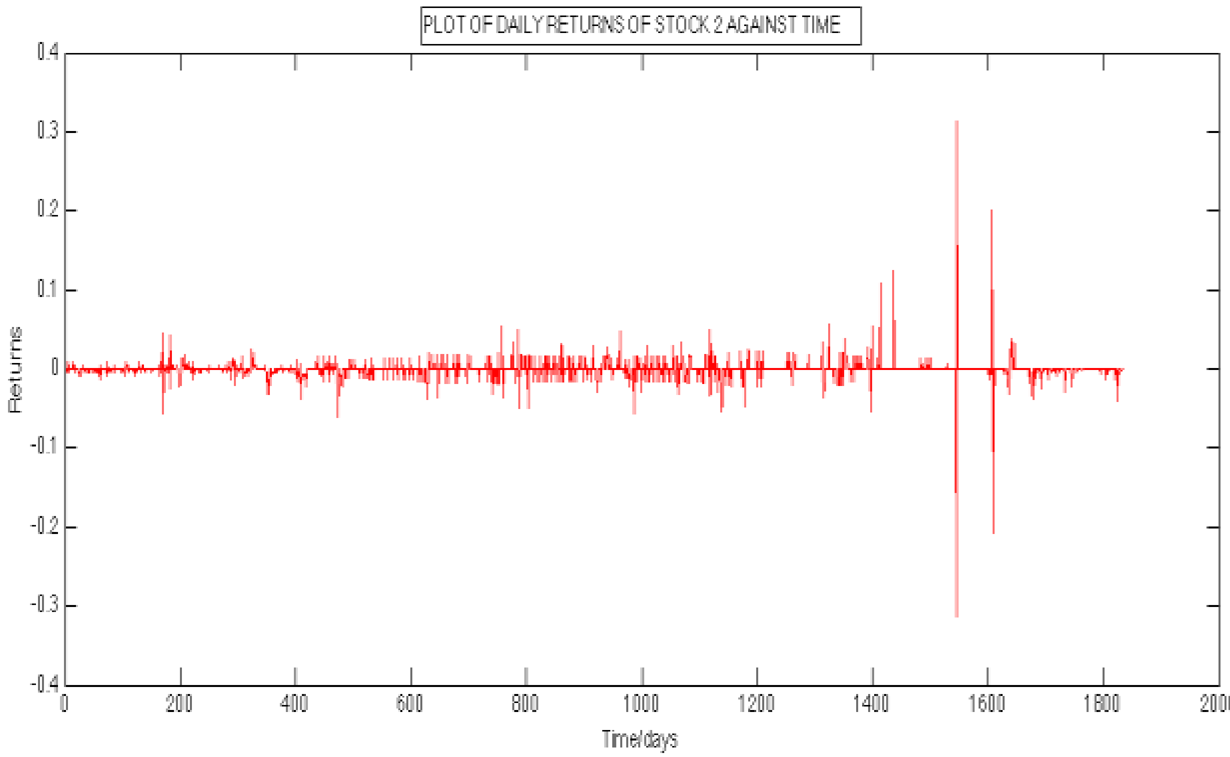


Figure 4.3: Line plot of returns returns of stock 2

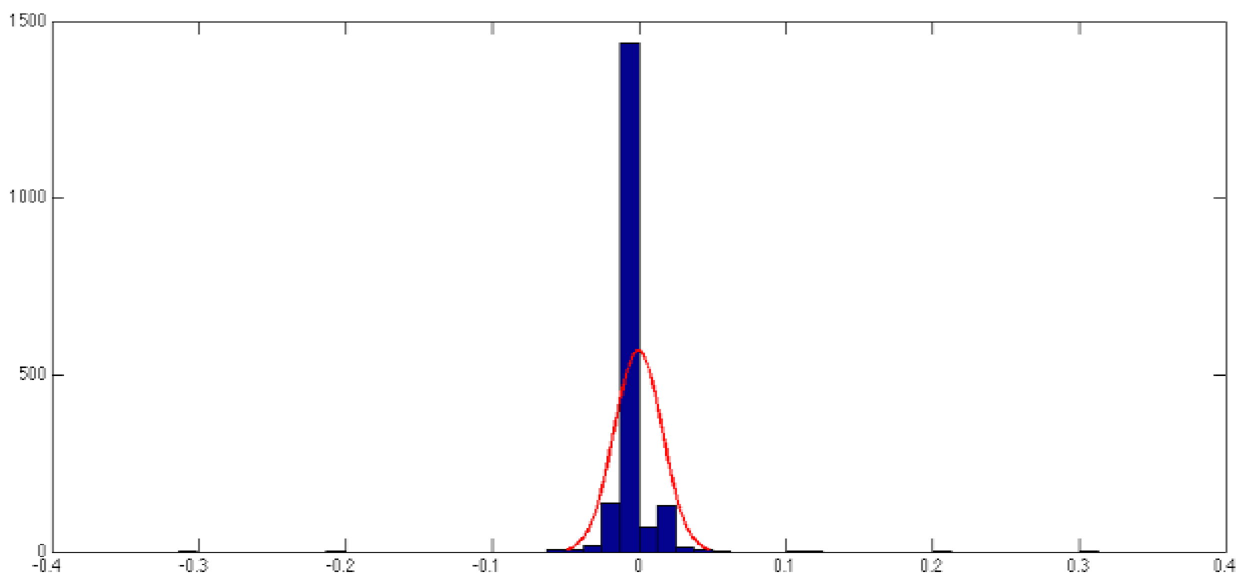


Figure 4.4:

Histogram showing the daily returns of Stock 2 combined with a normal distribution curve

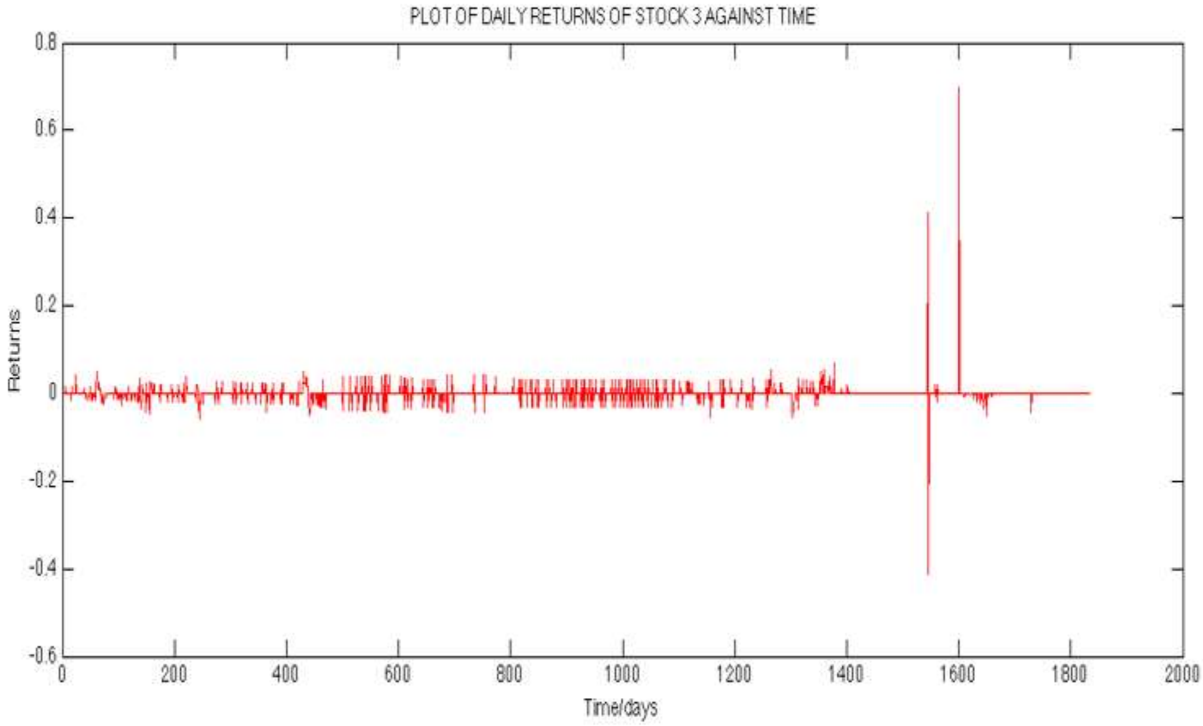


Figure 4.5: Line plot of returns returns of stock 3

Stock 5

With an annualized return of -0.000542, the average annualized volatility is 16.86438%. This is however associated with a standard deviation of 0.00109. From figure 4.9, it is clear that the returns of the stock actuated heavily as compared to the other five (5) stocks. The kurtosis is 2.04508. From the ($MATLAB$) function, the kurtosis of the normal distribution is 3 but distributions that are less outlier-prone have kurtosis less than 3, hence the distribution of returns for Stock 5 is less outlier prone. The skewness of Stock 5 returns is negatively skewed (-0.43877), indicating that the returns are skewed to the right. This is also a clear indication of the departure from the assumptions of normality in the parametric approach.

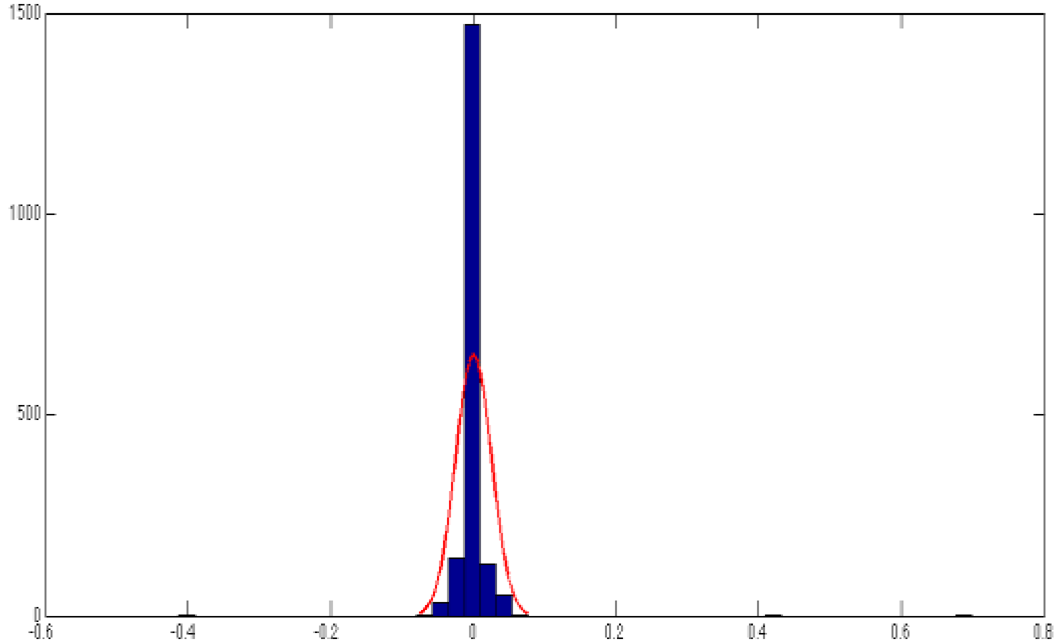


Figure 4.6: Histogram showing the daily returns of Stock 3 combined with a normal distribution curve

Stock 6

The annualized volatility of Stock 6 is 11.81406% and an annualized return of -0.000317. The kurtosis of Stock 6 is 1.97500. Skewness in figure 4.12 is 0.42958, meaning that the distribution is asymmetrical (lean to the right) with higher probability of positive values. Aside the kurtosis, the normal distribution curve is a pretty nice fit on these returns.

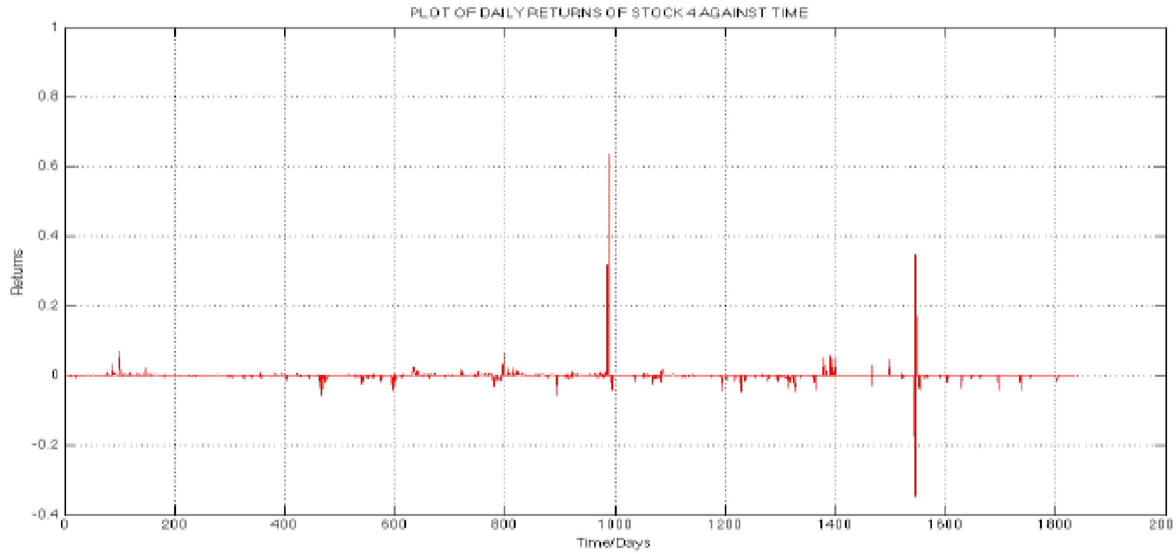


Figure 4.7: Line plot of returns returns of stock 4

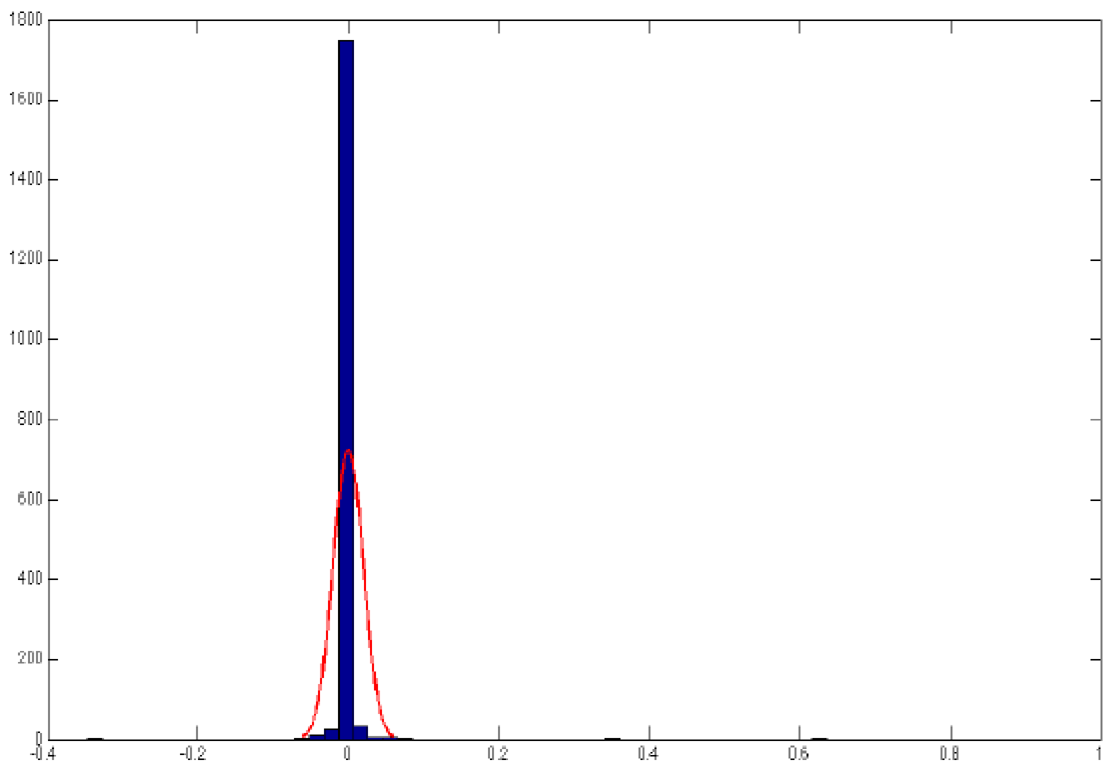


Figure 4.8:

Histogram showing the daily returns of Stock 4 combined with a normal distribution curve

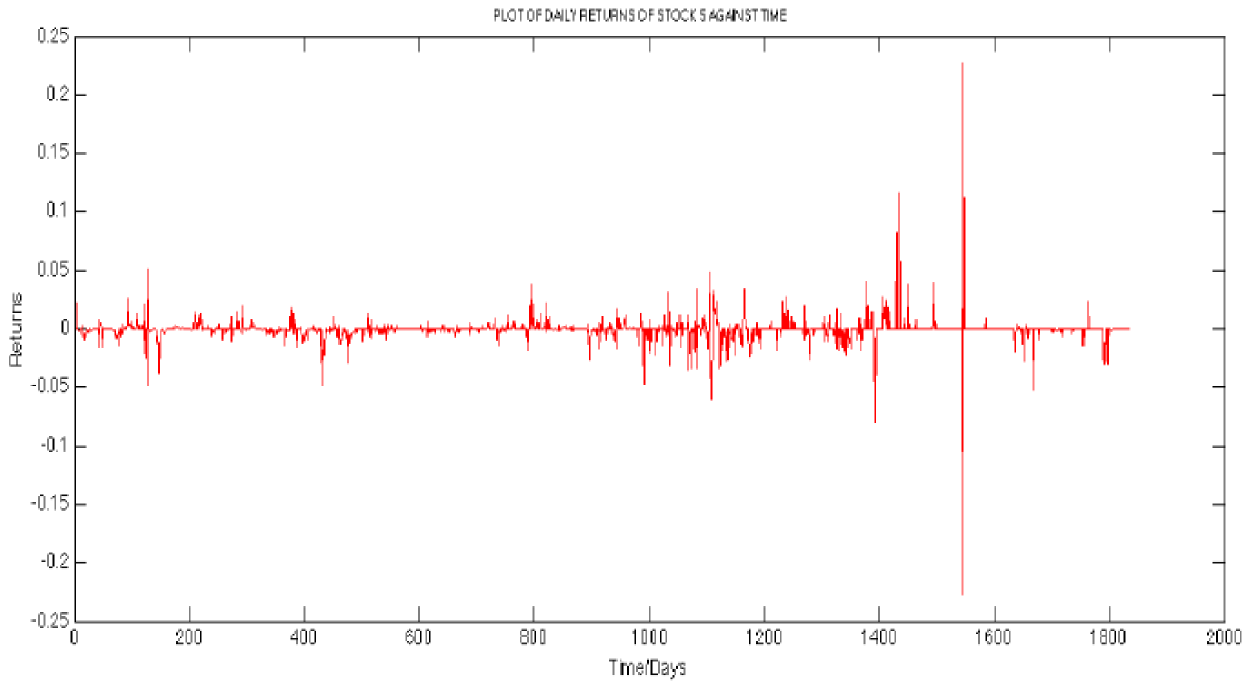


Figure 4.9: Line plot of returns returns of stock 5

4.4 Analysis

Experimental part of this studies refers to calculating VaR for portfolio using Historical simulation, Variance-Covariance approach, and Monte Carlo Approach. Portfolio consists of six (6) stocks listed on Ghana stock exchange. Each stock is assumed to have the same fixed proportion of $\frac{1}{6}$ in the portfolio and their total value is 1,000,000 Cedis.

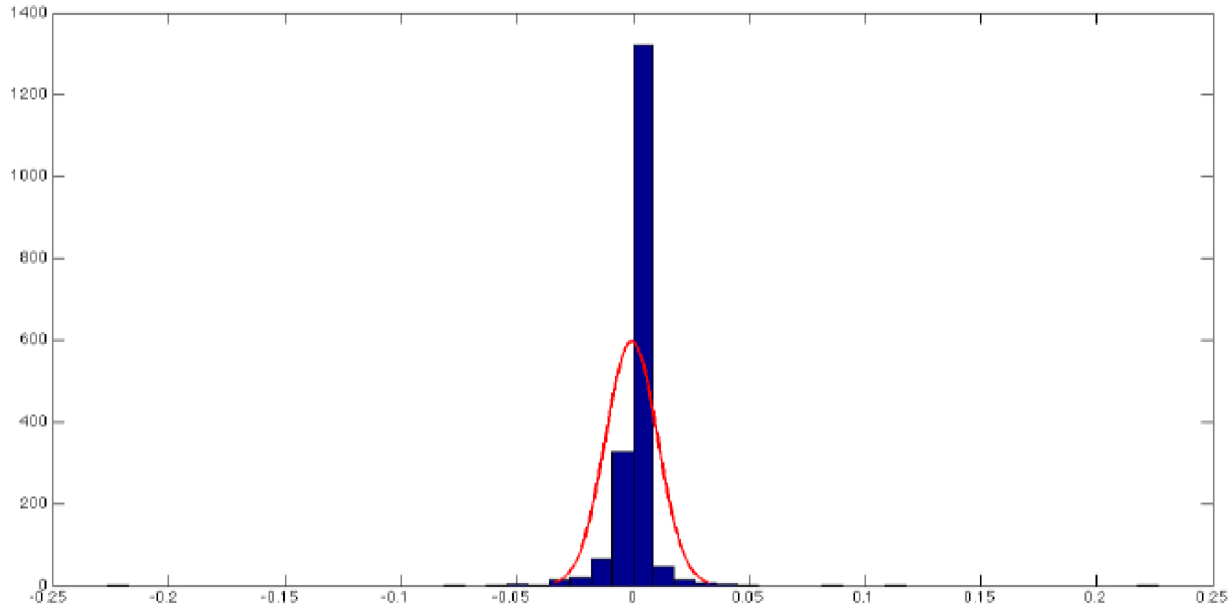


Figure 4.10:
Histogram showing the daily returns of Stock 5 combined with a normal distribution curve.

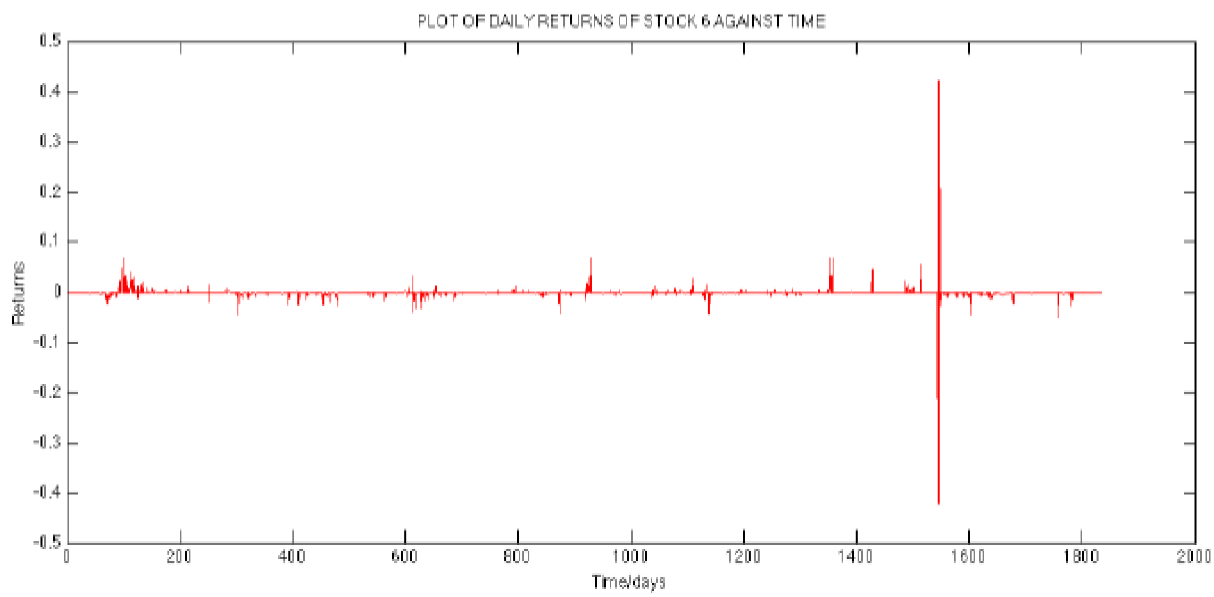


Figure 4.11: Line plot of returns returns of stock 6

Historical Simulation

Figure 4.14 shows the (*MATLAB*) output of the histogram of returns of the portfolio of stocks. The red bar indicates those returns in the portfolio that are less than the Value at

Risk Value. The Value at Risk of the portfolio of stocks for the historical simulation is -0.013161 with a simulation time of 0.1752 seconds. With a kurtosis of 592.1292, it is clear from Figure 4.14 that the distribution of returns of the portfolio of stocks from the historical simulation perspective is highly outlier prone. Also the high value of the kurtosis is because empirical distribution has higher peak than normal distribution and "bell" like shape which is narrower than the theoretical normal distribution. The distribution is positively skewed with skewness of 9.8684. The mean of the distribution of this approach is slightly lower than the normal distribution mean which is 0. With a change in confidence level from 95% to 99%, the Value at Risk changes from -0.013161 to -0.034382. The simulation time for this confidence level is 0.034382 seconds which is lower than that of the 95% confidence level. This indicates that it took less time to simulate this level of confidence.

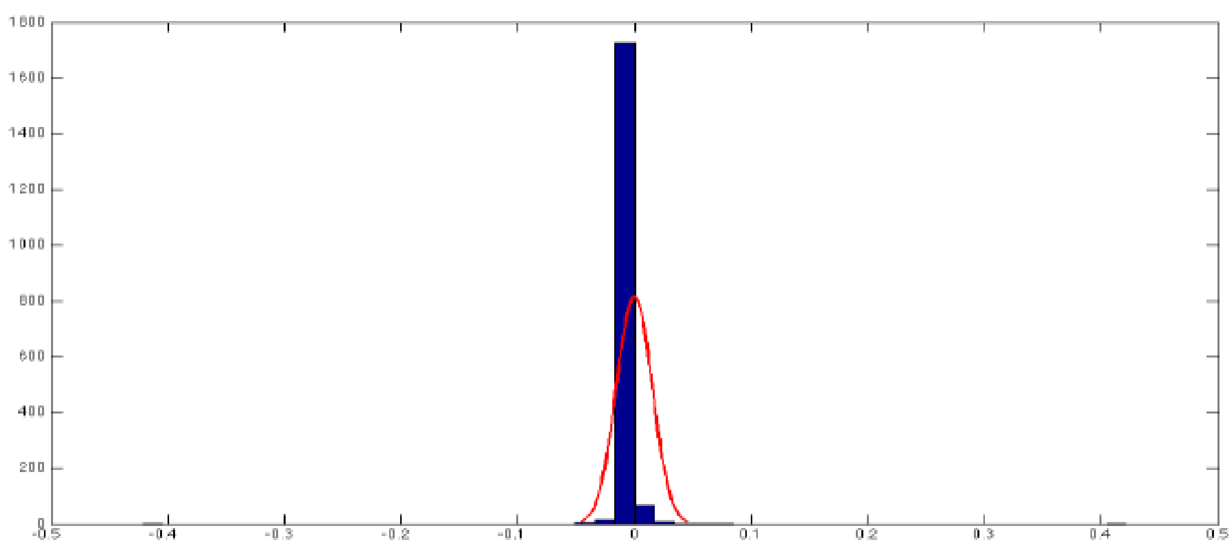


Figure 4.12: Histogram showing the daily returns of Stock 6 combined with a normal distribution curve.

4.4,2 Monte Carlo Simulation

Mean or expected return of the portfolio of stocks is slightly negative (-0.00024341) and the theoretical normal distribution has mean of 0. The kurtosis of the stocks portfolio is evaluated to be 3.0398, which is far lower than the kurtosis of the historical simulation. This is clear that even though the distributions in this approach may have outliers but it is not apart like the historical simulation. Figure 4.16 shows the ($MATLAB$) output of the histogram of returns of the portfolio of stocks. The red bar indicates those returns in the portfolio that are less than the Value at Risk Value. The Value at Risk of the portfolio of stocks at a 95% confidence level for the Monte Carlo simulation is -0.000515. The Value at Risk for a 99% confidence level for the portfolio of stocks in this simulation is -0.0006312 with an associated simulation time of 0.1785.

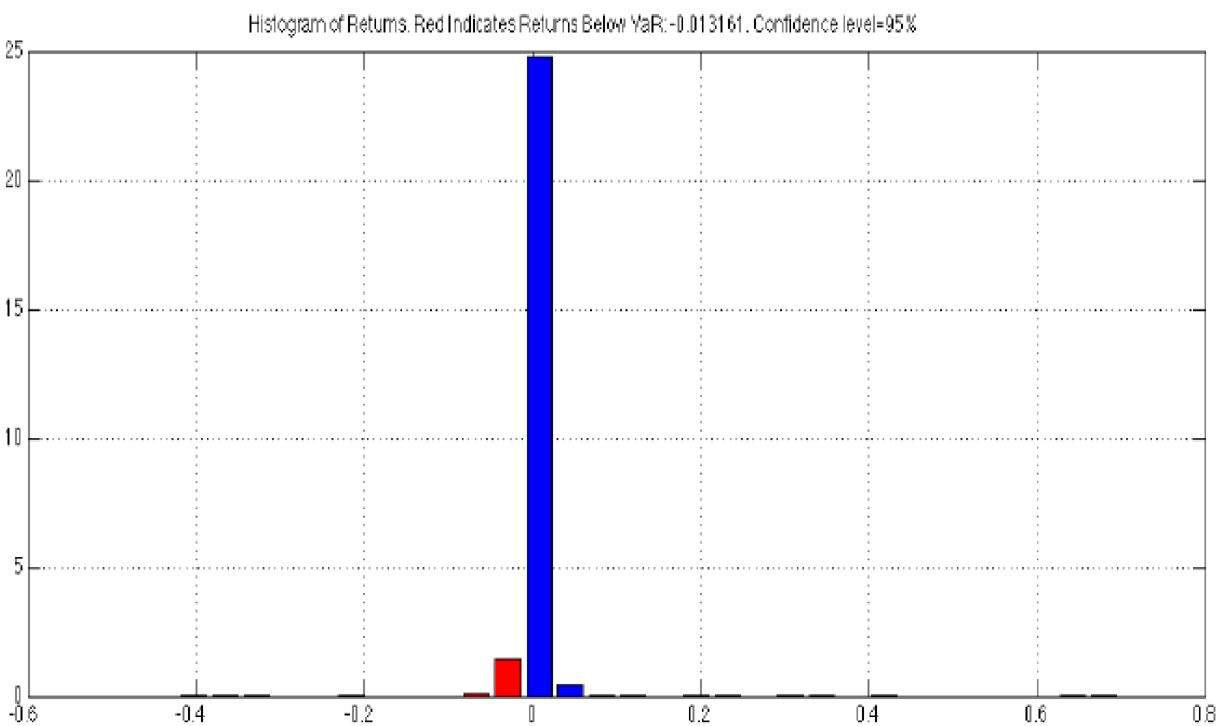


Figure 4.13: Histogram of VaR (95% Confidence level) for Historical Simulation

4.4.3 Comparison of Results Based on Simulation Time and VaR Values

Table 4.3 shows the time in seconds for simulation of the two approaches and their associated level of confidence. It is clear that the VaR99% had the highest simulation time and VaR99% least time in simulation. This least time is associated with Monte-Carlo simulation.

Figure 4.4 also shows the VaR values for the two different confidence levels and the two different approaches. From the table, Monte-Carlo Simulation had an optimal values of VaR as compared to Historical simulation.

	Historical Simulation Approach	Monte- Carlo Approach
$VaR_{99\%}$	0.1785	0.16565
$VaR_{95\%}$	0.1830	0.1752

Table 4.3: Simulation time of VaR at different levels of confidence

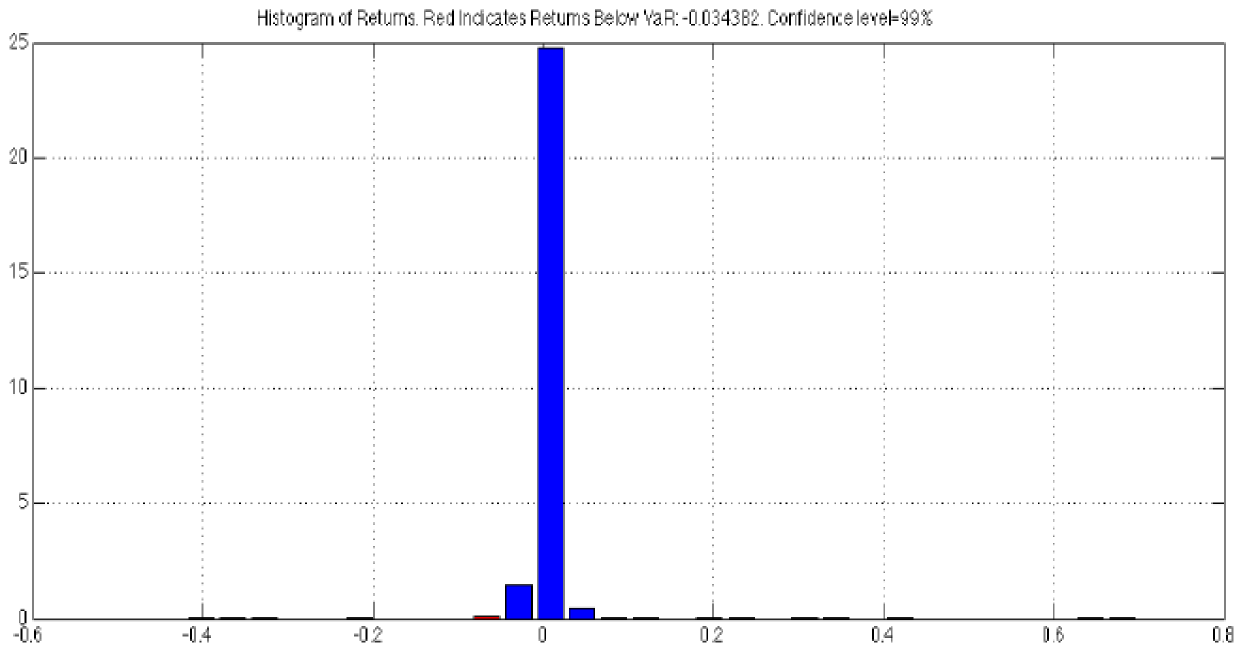


Figure4.14:Histogram of VaR(99% Confidence level) for Historical Simulation

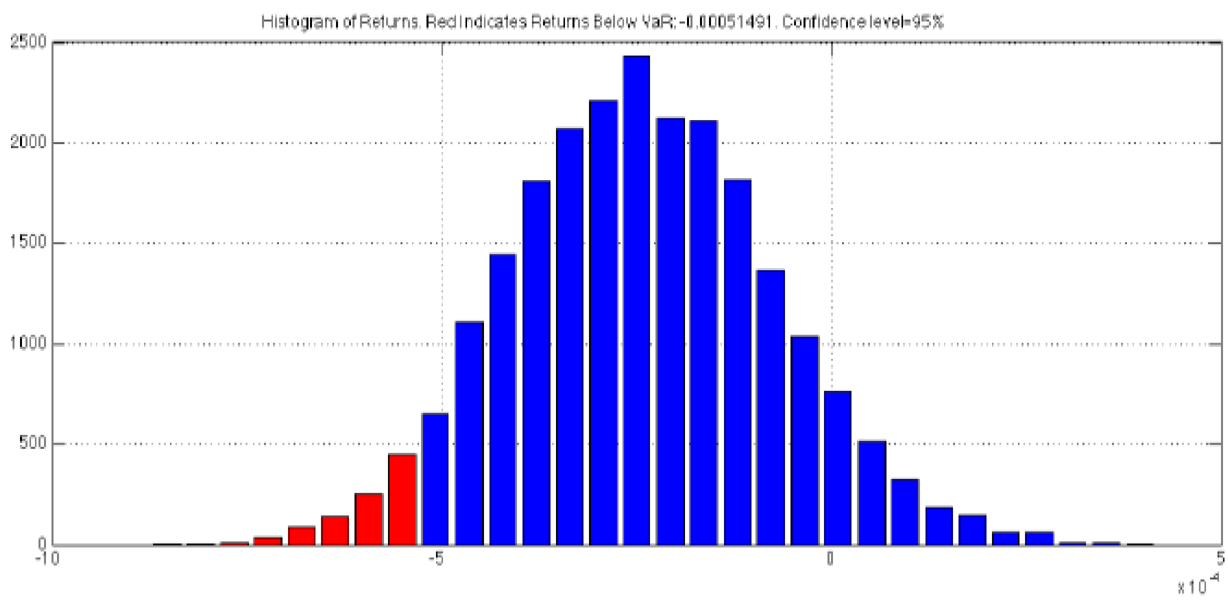


Figure 4.15: Histogram of VaR (95% Confidence level) for Monte-Carlo Simulation

	Historical Simulation Approach	Monte- Carlo Approach

$VaR_{99\%}$	-0.034382	-0.0006312
$VaR_{95\%}$	-0.013161	-0.00051419

Table 4.4: VaR values at different levels of confidence

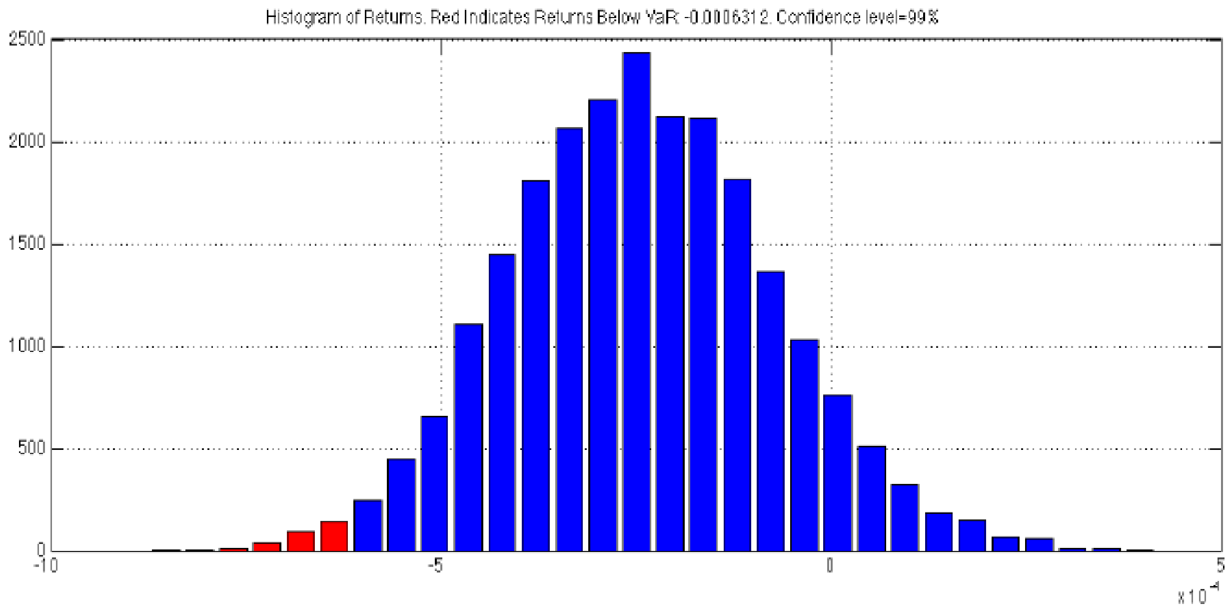


Figure 4.16: Histogram of VaR (99% Confidence level) for Monte-Carlo Simulation VaR as compared to Historical simulation

Chapter 5

Conclusions and Recommendations

This chapter accounts for the conclusions that the study has reached and suggests topics for future research.

5.1 Conclusion

From our analysis, Monte-Carlo simulation had an optimal value of VaR as compared to Historical simulation in both VaR95% and VaR99% confidence levels. From table 4.3, Monte-Carlo approach had the least time in simulation as compared to the Historical simulation which had the highest simulation time. Hence the Monte-Carlo simulation is the optimal approach for calculating VaR as compared to Historical simulation.

5.2 Recommendation

Based on our work, we recommend that investors should use Monte-Carlo approach in calculating VaR. For further studies the VaR calculation should be back tested and supplemented with stress testing to see what VaR will be exceeded in case of extreme (unusual) price changes.

References

Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1999).Coherent measures of risk. Mathematical Finance, 9(3), 203.

Basle Committee on Banking Supervision, 1996, Amendment to the capital accord to incorporate market risk.

Dowd, K. (1998). Beyond value at risk: The new science of risk management. Chichester: Wiley.

Dowd, K. (August, 2009). The Extreme Value Approach to VaR -An Introduction to Financial Engineering News, Issue 11.

Fama, F. E. & French, R. K. (1993). Common risk factors in the returns on stocks and bonds, Journal of Financial Economics, 33: 3-56.

Fama, F. E. & French, R. K. (1996). Multifactor Explanations of Asset pricing Anomalies, The Journal of Finance, 51(1): 55-84.

Goorbergh van den, R. and Vlaar, P., (1999a). Value-at-Risk Analysis of Stock Returns Historical Simulation, Variance Techniques or Tail Index Estimation?, De Nederlandsche Bank Staff Reports, 40.

Hendricks, D. (1996). Evaluation of Value-at-Risk models using Historical Data, Federal Reserve Bank of New York Economic Policy Review, 2(4): 39-70.

Hull, J. C. (1993). Options, Futures, and Other Derivative Securities, 2nd Edtion.

Engle wood Cliffs, NJ: Prentice Hall.

Hull, J., and White, A. 1998. Incorporating volatility updating into the historical simulation method for VaR, Journal of Risk, 1: 5-19.

Hull, J. (1997). Options, Futures, and Other Derivatives, Prentice-Hall Int.

Hull, J. (2002). Fundamentals of Futures and Options Markets. Prentice Hall, 4th edition

JPMorgan/Reuters (1996), RiskMetrics - Technical Document. JPMorgan/Reuters.

Jorion, P. (June 1995). Predicting Volatility in the Foreign Exchange Market. Journal of Finance, 507-528.

Jorion, P. (2000). Value at Risk: The New Benchmark for Managing Financial Risk, McGraw-Hill Professional.

Jorion P. (2001). Value at Risk: The new benchmarking for managing financial risk, 2nd, McGraw- Hill Trade.

Jorion, P. (2007). Value at Risk: The new benchmark for managing financial risk, London: McGraw-Hill.

Lopez, J. A. (1998). Methods for evaluating Value-at-Risk estimates. Federal Reserve Bank of New York, Economic Policy Review financial time series: An extreme value approach, Journal of Empirical Finance,7: 271-300.

Linsmeier, Thomas J. and Neil D. Pearson (1996). Risk Measurement: An Introduction to Value at Risk, working paper, Office for Futures and Options Research, University of Illinois, Mandelbrot, B. (1963). The variation of certain speculative prices. Journal of Business, 36, 394-419.

Manganelli, S., & Engle, R. F. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles. Journal of Business & Economic Statistics. Markowitz, H., 1952, Portfolio Selection, Journal of Finance, 7 (1), 77-91.

Sarma, M.,Thomas S., and Shah., A. (2003). Selection of VaR models, Journal of Forecasting, 22 (4): 337-358.

Sharpe, W. F. (2000). Portfolio Theory and Capital Markets. McGraw-Hill (Originally published in 1970.).

Taleb, N. (1997). The world according to Nassim Taleb. Derivatives Strategy.

Whalen, C (2006). Basel II Comment Letter. (accessed April 15, 2015).

Sharpe, W. F. (2000). Portfolio Theory and Capital Markets. McGraw-Hill (Originally published in 1970).

Appendix A

Code 1

```
function createfigure (VertexNormals1, YData1, XData1, Vertices1,....

Faces1, FaceVertexCData1, CData1, X1, Y1)

% Create figure

figure1 = figure;

% Create axes

axes1 = axes('Parent',figure1,'CLim',[1 2]);

box(axes1,'on');

hold(axes1,'all');

% Create patch

patch('Parent',axes1,'VertexNormals',VertexNormals1,'YData',YData1,...

'XData',XData1,...

'Vertices',Vertices1,...

'Faces',Faces1,...
```

```
'FaceColor','flat',...

'FaceVertexCData',FaceVertexCData1,...

'CData',CData1);

% Create plot

plot(X1,Y1,'LineWidth',2,'Color',[1 0 0]);
```

Code 2

```
function VaR = jones(returns, confidence _level, plot_ flag)
% Inputs:

% returns Vector of returns

% confidence _level Confidence level (default 0.95)

% plot _flag if true, visualize result (default is true)

%

% Outputs:

% VaR Value at Risk

% handle inputs

tic

if nargin < 3

plot _flag = true;

end

if nargin < 2

confidence _level = 0.95;

end
```

```
% Sort returns from smallest to largest

Sorted _returns = sort (returns);

% Store the number of returns

  Num _returns = numel (returns);

% Calculate the index of the sorted return that will be VaR

VaR _index = ceil((1-confidence_level)*num _returns);

% Use the index to extract VaR from sorted return
```