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Comparing Historical Simulation and Monte Carlo Simulation in Calculating VaR

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Abstract

Since the publication of JP Morgan's Risk Metrics in 1994 there has been an explosion in the research in the areas of value of risk and risk management in general. While the fundamental ideas encompassing VaR are founded in the area of market risk measurement they have been reached out throughout the most recent decade, to different territories of risk management. Specifically, VaR models are presently usually used to gauge both credit and operational risks. In any case, with different methods and models, the decision that VaR users face is the decision of selecting the proper procedure that is generally suitable. The strategies ought to make gauges that fit the normal conveyance of returns. On the off chance that VaR is overestimated, the administrators winds up overestimating the danger. This, in any case, could bring about the holding of great measures of cash to cover misfortunes as for the situation with banks under the Basel II accord, (Basle Committee on Banking Supervision, 1996). The same goes for the inverse occasion, when VaR has been thought little of bringing about inability to cover acquired losses. This study seeks to compare two different methods to calculating VaR namely Historical Simulation and Monte Carlo Simulation. The method will be applied on six different equities on the Ghana Stock Exchange Market with two different confidence level of 95% and 99%.

Keywords: *VaR, Historical Simulation and Monte Carlo Simulation*

I. INTRODUCTION

Researchers in the field of financial Mathematics and Economics have long identified the significance of measuring the risk of a portfolio of financial assets or securities. Vehemently, concerns go back at least forty years, when Markowitz's earth shattering work on portfolio choice (1959) investigated the suitable definition and estimation of danger. In the field of investment, risk is a measure of how unstable assets returns are. Introduction to this instability can prompt a misfortune in one's ventures. Thus instruments are utilized not just to latently measure and report risk, but to protectively control or effectively oversee it. Notwithstanding, a system progressed in writing includes the utilization of Value-at-Risk (VaR) models.

The idea and utilization of value at risk is recent. Value at risk was first utilized by major financial firms in the late 1980's to quantify the dangers of their exchanging portfolios. Right now Value at risk was utilized by most major derivative dealers in remote nations to gauge and oversee market hazard. It is additionally progressively being utilized by smaller financial organizations, non-financial organizations, and institutional investors. A *VaR* model measures market risk by deciding how much the estimation of a portfolio could decrease over a given timeframe with a given likelihood as an aftereffect of changes in interest rates, foreign exchange rates, equity prices, or commodity prices.

For instance, if the given timeframe is one day and the given likelihood is 1 percent, the *VaR* measure would be an evaluation of the decrease in the portfolio esteem that could happen with a 1% likelihood throughout the following exchanging day. At the end of the day, if the *VaR* measure is exact, losses greater than the *VaR* measure ought to happen under 1% of the time. *VaR* models total the several components of price risk into a single quantitative measure of the potential for losses over a specified time horizon. These models are obviously engaging in light of the fact that they pass available risk of the whole portfolio in one number.

There are diverse procedures to compute the *VaR*, most popular are Historical simulation, Monte-Carlo simulation. Variance-Covariance, J. P. Morgan's Risk Metrics System. For investors, danger is about the chances of losing cash, and VaR depends on that common- sense fact. By accepting that investors think about the chances of enormous misfortunes, VaR can be utilized to answer the inquiries. The *VaR* measurement has three parts: a period, a certainty level and a loss amount (or loss percentage). It can thus be used to answer question such as: "What is the most I can (with a 95% or 99% level of confidence) expect to lose in Cedis over the next month"?, "What is the maximum percentage I can (with 95% or 99% confidence) expect to lose over the next year”?, ”What is my worst-case scenario”? or ‘’How much could I lose in a really bad month”?

II. VALUE AT RISK (VAR)

An exact computation of risk is a crucial first step for real risk management, and financial mediators, because of the nature of their business, tend to be leading developers of new risk measurement techniques. In the past, many of these models were internal models, developed in-house by financial organizations. As a matter of fact, the VaR tool is complementary to many other internal risk measures. Nevertheless, market forces during the late 1990s established conditions that led to the development of VaR as a main risk measurement tool for financial firms.
“how much can we lose on our trading portfolio by tomorrow’s close?”

The above question was made by Dennis Weatherstone, who was at the time the Chairman of JP Morgan. There are two approaches in answering Weatherstone’s question. The first is a probabilistic/statistical approach which is the center of the VaR measure and the other is the scenario approach-an event-driven, non-quantitative, subjective approach, which computes the effect on the portfolio value of a scenario or a set of scenarios that indicate what is considered adverse circumstances. VaR takes a probabilistic or statistical approach to answering Mr. Weatherstone’s question of how much could be lost on a “worst day.” Hence, the definition of “worst day” in a statistical sense, such that there is only a *y* percent probability that daily losses will run over this amount given a distribution of all feasible daily returns over some current past period. Therefore, we define a “worst day” so that there is only a *y* percent probability of an even worse day.

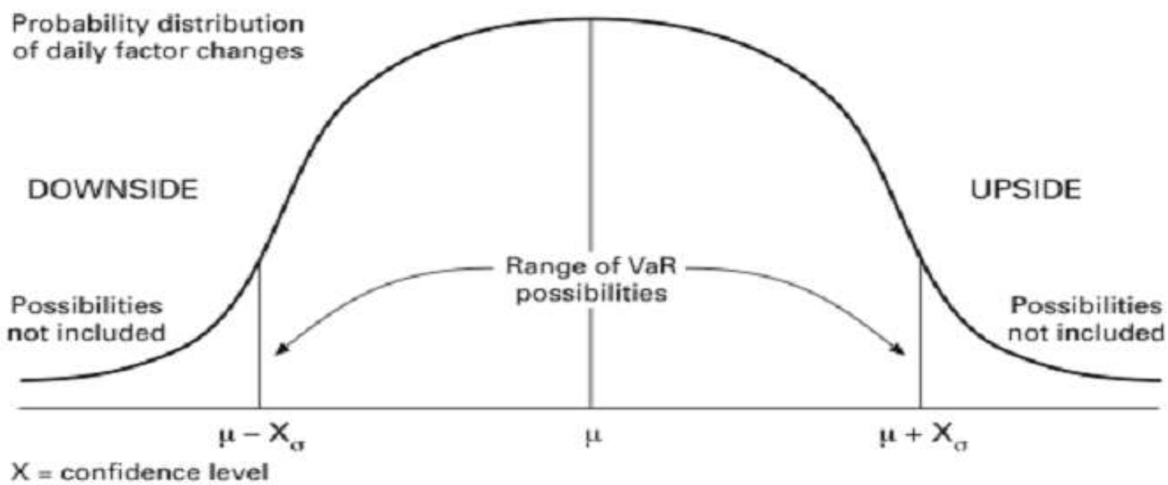

Figure 3.1: VaR and the normal distribution

A. Definition of Value-at-Risk

A value-at-risk model evaluates the market risk by determining how much the value of a portfolio could decrease over a given period of time with a given probability as a result of changes in market prices or rates. It allows managers and investors to say: *“we are X percent certain that we will not lose more than V Cedis in the next N days”* (Hull, 2002). The variable V is the Value at Risk. It is a function of two components and this components greatly affects the nature of the value-at-risk model.

- N (The time Horizon) - It is a period of time over which VaR is measured. It is traditionally measured in trading days rather than calendar days. Pragmatically, financial analysts mostly set $N = 1$, because of lack of data to estimate the behaviour of market variables over longer period of time.
- Y (The Confidence level) - Frequently used confidence levels are 99% and 95%. For instance, a 500 Cedis, one day, 95% confidence level VaR value for a stock means that during the next day we are 95% certain that

the value of our asset in this specific stock will not decrease by more than 500 Cedis. The VaR will decrease if a lower confidence level say 99% or 95% is chosen. Different confidence levels will suit different organizations and purposes and will be chosen according to financial analyst's relation to risk. The more risk averse the firm is the higher the confidence level will be selected.

To provide an overview of the measure of VaR, a simple example is given. Assume that the unit share price of Tullow oil on the Ghana Stock Exchange market today is 20 Cedis and the daily standard deviation (σ) is 4 Cedis. Investors that purchase larger shares might want to know how much, given a certain confidence level, they can possibly lose when purchasing the share today compared to tomorrow. Suppose, the chosen confidence interval is 99%, this means that a day out of hundred, the loss will be greater than the calculated VaR. This is true when the share price is normally distributed around the mean price change.

Value due to a decrease in the share price = $20 - 2.33 \text{ x } \sigma = 10.68$ Value due to an increase in the share price = $20 + 2.33 \text{ x } \sigma = 29{:}\ 32$
This can be explained as, with a 99% probability, the loss will not be greater than $20 - 10.68 = 9.32$ Ghana Cedis which is the VaR for a confidence level of 99%.

B. Assumptions behind Value-at-Risk

As often as possible, some measurable presumptions are made keeping in mind the end goal to compute the VaR. The stationarity prerequisite. That is, a 1% change in returns is similarly prone to happen anytime. Stationarity is a customary presumption in money related financial aspects, since it disentangles calculations significantly. A related presumption is the random walk assumption of inter-temporal unpredictability. That is, everyday varieties in returns are autonomous; say, a lessening in the Ghana Stock file on one day of *y*% has no prescient force concerning returns on the Ghana Stock record on the following day. Additionally, the random walk assumption can be depicted as the presumption of a normal rate of return equivalent to zero, as in the value portfolio sample. Henceforth, if the normal day by day return is zero, then the ideal speculation assessment of tomorrow's value level is today's level. A basic supposition is the non-negativity requirement, which obviously expresses that money related resources with limited obligation can't accomplish negative qualities. All things considered, subsidiaries (examples: forwards, futures, and swaps) can repudiate this presumption. The time consistency necessity declares that all unit period suppositions hold over the multi-period time horizon. Another most noteworthy suspicion is the distributional assumption. In the basic equity portfolio illustration, it can be expected that every day return varieties in the Ghana Stock file take after a typical dissemination. The supposition has the upside of making the VaR estimations much less demanding. In any case it has a few downsides. The value changes don't generally suit the typical appropriation bend and when more perceptions are found in the tails, ordinary based VaR will downplay the misfortunes that can happen.

C. Steps in calculating Value-at-Risk

There are three steps in VaR calculations. The first step is the holding period, the time period over which the losses may occur. This period is mostly a day, however it can be more or less conditional on a particular situation. Investors who actively trade their portfolios has the tendency to use a 1-day holding period, whereas longer holding periods are more pragmatic for nonfinancial firms and institutional investors. The longer the holding period, the larger the VaR.
The next step is the probability of losses more than VaR, p, needs to be stated, with the most ordinary probability level being 1%. Literature provides little direction about the choice of p; it is mainly decided by how the user of the risk management system desires to explain the VaR number. VaR levels of 99% to 95% are ordinary in practice, however less extreme higher numbers are often used in risk control on the trading floor and most extreme lower numbers may be used for applications like survival analysis, economic capital, or long-run risk analysis. The last step is to determine the probability distribution of the profit and loss of the portfolio. This is the most problematic and significance aspect of risk modeling. The normal practice is to evaluate the distribution by using historical observations and a statistical model. Calculate the VaR estimate - this is accomplished by observing the loss amount related with that area under the normal curve at the critical confidence interval value that is statistically related with the probability chosen for the VaR estimate in step 2.

D. Interpreting and Analyzing Value-at-Risk

In deciphering and looking at VaR numbers, it is basic to give careful consideration to the likelihood and holding period since, without them, VaR numbers are valueless. Case in point, a portfolio containing the same resource could deliver two divergent VaR estimates if risk managers choose different values of the probability of losses more than VaR and holding periods. Plainly, a loss permitted with a likelihood of just 3% rises above a loss permitted with a likelihood of 7%. With regards to the VaR of an organization's arrangement of positions is a related measure of the risk of budgetary anguish over a brief period depend on the liquidity of portfolio positions and the risk of amazing money outpourings. Antagonistic liquidity conditions lead to high exchange costs, for example, wide spreads and large margin calls. VaR is unrealistic to catch these effects. In danger enhancement, VaR is an imperative stride forward as for customary measures in view of susceptibilities to market variables. VaR is a complete thought and can be actualized to most money related instruments. It encapsulates in a single number all the risks of a portfolio fusing loan cost hazard, remote trade hazard. It additionally speeds up examinations between disparate resource classes. The VaR measure consolidates quantile (loss) and likelihood.

E. Measuring Returns

From the definition of VaR, the VaR number is the portfolio return in the worst case, hence the definition of portfolio return is first introduced. The return on Portfolio, ΔP is the difference between portfolio values, that is, $\Delta P = Pt+1 - Pt$, where the portfolio values at time t and $t + 1$ are Pt and $Pt+1$ respectively. The portfolio returns can be described by the rate of return. Nonetheless, there are two kinds of rates, namely the arithmetic and geometric. Geometric rate of r e y u u rn R_g is the logarithm of t h e price ratio, mathematically, $R_g = \ln\frac{P_t}{P_{t-1}}$ whilst arithmetic rate of return R_α, is portfolio return divides the original value , mathematically $R_\alpha = \frac{P_t - P_{t-1}}{P_{t-1}}$

By substitution, it can be noted that $R_g = \frac{P_t - P_{t-1}}{P_{t-1}} = \ln(1+R_\alpha)$. Suppose the time horizon is short, the daily arithmetic rate of return is around zero, then by Taylor expansion,

$$R_g = R_\alpha - \frac{R_\alpha^2}{2} + \frac{R_\alpha^3}{3} - \frac{R_\alpha^4}{4} + \ldots$$

Hence $R_\alpha \cong R_g$, implying that arithmetic rate and geometric rate are the same and we can use R to denote both. Let $R_{t,n}$ be the rate of return during the last n days, by geometric rate of returns:

$$R_{t,n} = \ln(\frac{P_t}{P_{t-1}}) = \ln(\frac{P_{t-1}}{P_{t-2}}) + \ldots + \ln(\frac{P_{t-n+1}}{P_{t-n}}) = R_t + R_{t-1} + \ldots + R_{t-n+1}$$

The rate of return during the last n days is the sum of n proceeding rates. The portfolio return of two consecutive days is $R_{t,2} = R_t + R_{t-1}$.

F. Deriving Value-at-Risk

The loss on a trading portfolio such that there is a probability p of losses equaling or exceeding *VaR* in a given trading period and a (1 - p) probability of losses being lower than the *VaR*. Mostly written as VaR (p) or *VaR* $100 \times P\%$ making the reliance on probability clear for instance, *VaR* (0.01) or *VaR* 1%. Probability levels mostly used in calculating *VaR* is 99% and 99.5%, however, percentage values that are lower and higher than these are mostly used in its application. *VaR* is a quantile on the distribution of Profit and Loss ($P = L$). Let the random variable R denote the ($P = L$) on an investment portfolio, with a specific realization say r. If one unit of an asset is held, ($P = L$) can be written as:

$$R = P_t - P_{t-1} \qquad (3.1)$$

Comprehensively, if the portfolio value is ψ and the returns is Y, then:

$R = \psi Y$ (3.2)

Let the density of P/L be denoted by *fr* (.), then VaR is given by:

$$\Pr\left[R \le -VaR(p)\right] = p \qquad (3.3)$$

$$p = \int_{-\inf}^{-VaR(p)} f_r(y)dy \qquad (3.4)$$

From equations (3.1) - (3.4), *VaR* can be derived from simple returns,

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

Assuming the return is zero and volatility is indicated by σ, then from the definition of *VaR* in (3.3) and (3.4), *VaR* can be obtained from:

$$p = P_r(P_t - P_{t-1} \le -VaR(p))$$

$$p = P_r(P_{t-1} - R_t \le -VaR(p))$$

$$p = P_r(\frac{R_t}{\sigma} \le -\frac{VaR(p)}{P_{t-1\sigma}})$$

Let the distribution of standardized returns $\frac{R_t}{\sigma}$ be *QR*(.) and the distribution by *Q*−1(*p*).
Hence the VaR for holding a unit of the asset is:

$$VaR(p) = -\sigma Q_R^{-1}(p)P_{t-1}$$

The significance level can be denoted by $y(p) = Q_R^{-1}(p)$ the VaR equation can be written as:

$$VaR(p) = -\sigma y(p)P_{t-1}$$

However if continuously compounded returns are used:

$$Y_t = \log(1 + R_t) = \log(\frac{P_t}{P_{t-1}})$$

$$Y_t = \log P_t - \log P_{t-1}$$

This implies that,

$$p = P_r(P_t - P_{t-1} \le -VaR(p))$$

$$p = P_r(P_{t-1}(e^{Y_{t-1}}) \le -VaR(p))$$

$$p = P_r(\frac{R_t}{\sigma} \le \log(-\frac{VaR(p)}{P_{t-1}} + 1)\frac{1}{\sigma})$$

Since $-\frac{VaR(p)}{P_{t-1}} \le 1$ we can denote the distribution of standardized returns $\frac{Y_t}{\sigma}$ by *Qy* (.) the inverse distribution by $y(p) = Q_y^{-1}(p)$, we have

$$VaR(p) = (-\exp(Q_y^{-1}(p)\sigma) - 1)P_{t-1}$$

and for small $Q_y^{-1}(p)\sigma$, the *VaR* for holding one unit of the asset is given by:

$$VaR(p) \approx -\sigma y(p) P_{t-1}$$

So, the *VaR* for continuously compounded returns is approximately the same as the *VaR* using simple returns.

III. COHERENCE

The properties a risk measure should have in order to be considered a functional risk mea- sure was studied by Artzner et al. (1999); they determined four axioms that risk measures ideally should comply with. If a risk measure satisfies these four axioms it is called coherent. Let τ denote a risk measure. In this work, our risk measure is the VaR.

A. Definition

Consider two real-valued random variables: A and B. A function $\tau(.)$: $A, B \rightarrow \Re$ is called a coherent risk measure if it satisfies for A, B and constant k. V is the VaR.

1. *Subadditivity*

$A, B, A + B \in V \Rightarrow \tau(A + B) \leq \tau(A) + \tau(B)$

The risk to the portfolios of A and B cannot be worse than the sum of the two individual risks-an illustration of the diversification principle.

2. *Translation Invariance*

$A \in V, k \in \Re \Rightarrow \tau(A + k) = \tau(A) - k$

Adding k to the portfolio is like adding cash, which acts as insurance, so the risk of

$A + k$ is less than the risk of A by the amount of cash, k.

3. *Positive Homogeneity*

$A \in V, k > 0 \Rightarrow \tau(kA) = k\tau(A)$ for $k > 0$

For instance, if the portfolio value doubles ($k = 2$) then the risk doubles.

4. *Monotonicity*

$A, B \in V, A \leq B \Rightarrow \tau(A) \geq \tau(B)$

If portfolio A never transcends the values of portfolio B (that is, it is always more negative, consequently, its losses will be equal or larger), the risk of B should never surpass the risk of A. However the axiom of positive homogeneity is pragmatically violated. For instance, suppose the maximum risk a portfolio worth five hundred Ghana Cedis can hold is thirty Cedis. Then this implies from a axiom 3 that, whenever the portfolio is doubled, the risk should also be doubled. But this is always not the case because as relative shareholdings increase and/or the liquidity of a stock decreases, risk may increase more rapidly than the portfolio size. In such a situation positive homogeneity is violated and: $\tau(kA) > k\tau(A)$ (3.5)

Among these four axioms, the most important is the sub-additivity. A portfolio of assets is measured as less risky than the sum of the risks of distinct assets if this axiom holds. If VaR violate this axiom, it can erroneously be concluded that diversification results in an increase in risk. VaR is sub-additive in the special case of normally distributed returns. Dan´ielsson et al. (2010a) studied the sub-additivity of VaR in details and found out that VaR is actually sub-additive conditioned that the tail index exceeds 2- when the second moment, or variance, is defined under a condition of multivariate regular variation.

IV. HISTORICAL SIMULATION (HS) METHOD

Historical simulation can also be used in estimating the Value at Risk. Historical Simulation is more pliable than the Parametric method and avoids some of the pitfalls of the parametric method. This method has the benefit of simply handling options in the portfolio (Best, 1998). It also has the benefit of extensively accepted by trading communities and management mostly because of its clarity. The historical simulation method calculates potential losses using real historical data of the returns in the risk factors and hence captures the non-normal distribution of risk factor returns. Because the risk factor returns used for revaluing the portfolio are real past movements, the correlation in the estimation are also actual historical correlations. As Dan´ ielsson (2011) clearly stated, the main concept of this methodology is to predict future losses based on the historical performance. Historical simulation (HS) is a simple method for forecasting risk and relies on the assumption that history repeats itself, where one of the observed historical returns is anticipated to be the next period return. Each historical observation carries the same weight in HS forecasting. This can be a disadvantage, specifically when there is a structural break in volatility. Nevertheless, in the absence of structural breaks, HS tends to function better than alternative methods. It is less sensitive to the odd outlier and does not absorb estimation error in the same way as parametric methods. The importance of HS become especially clear when working with portfolios because it directly captures nonlinear dependence in a way that other methods cannot.

Values of the market components for a specific past period are fetched and changes in these values over the time horizon are observed for use in the calculation. For example, if a 1-day VaR is needed using the past 50 trading days, each of the market factors will have a vector of observed changes that will be made up of the 49 changes in value of the market factor. A vector of different values is generated for each of the market factors by adding the contemporary value of the market factor to each of the values in the vector of observed changes.

The portfolio value is constructed using the present and alternative values for the market factors. The variations in portfolio value between the recent value and the alternative values are then evaluated. The last step is to categorize the changes in portfolio value from the smallest value to highest value and ascertain VaR based on the required confidence interval. For a 1-day, 95% confidence level VaR using the past 100 trading days, the VaR would be the 95th most unfavourable change in portfolio value.
The risk is calculated with price changes:

5. Absolute change in price,
6. Logarithmic change in price,
7. Relative change in price, but should the change be relative to the initial price, then it is called return or rate of return.

1-day Period

The price in time t can be denoted as *Pt* (which represents one trading day). The relative rate of return (*Rt*), between *t* and *t* − 1 can be calculated as:

$$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

The logarithmic rate of return (*Rlt*) correspond to

$$(Rl_t) = \log \frac{P_t}{P_{t-1}} = \log(1 + R_t)$$

The absolute rate of return (*Rat*) for the same time period is

$$Ra_t = P_t - P_{t-1}$$

K- days Period

Return of the k-days period of time is defined as

$$R_t = \frac{P_t - P_{t-1}}{P_{t-k}}$$

The main assumptions of HS are:

- Selected sample period could describe the properties of assets very well,
- There is a probability of reiterating the past in the future, that is, the recreation of

the patterns appeared in the volatilities and correlations of the returns in historical sample, in the future. However, the past could be a good basis of the future forecast.

The process used to estimate the VaR of a given portfolio using historical simulation is as below:

1. A portfolio of M assets denoted by a vector of weights is defined;

$$\bar{\omega} = \begin{pmatrix} \omega_0, 1 \\ \omega_0, 2 \\ \omega_0, 3 \\ \vdots \\ \omega_0, M \end{pmatrix} \tag{3.6}$$

2. For each asset price or risk factor involved in the problem, obtain a series of returns for a given time period (for example, 200 days). When log-returns are used, they are

calculated as below:

$$r_{k,t} = \log \frac{pk, t}{pk, t-1} \tag{3.7}$$

where $r_{k,t}$ and pk, t are respectively the return and price of the asset *k* at time *t*.

3. Consider each of the days in the time series of returns as a scenario for possible Changes in the next day. As there are M assets, each day t of historical data will form a scenario defined by:

$$\bar{r}_t = \begin{pmatrix} r_1, t \\ r_2, t \\ r_3, t \\ \vdots \\ \omega M, t \end{pmatrix} \tag{3.8}$$

It is important to notice that from this point on the scenarios $\bar{r}_t$ are no longer seen as time series, but just as a set of different possible realizations of the random vectors $\bar{r}_t$, obtained from historical data.

4. Apply each of the scenarios to the composition of the portfolio today, that is, do not apply the price changes in cascade to the portfolio. Indicating that the outcome of the application of scenario t to the portfolio is:

$$\bar{\omega} = \begin{pmatrix} \omega_0, 1 \\ \omega_0, 2 \\ \omega_0, 3 \\ \vdots \\ \omega_0, M \end{pmatrix} = \begin{pmatrix} \omega_0, 1.e^{r1,t} \\ \omega_0, 2e^{r2,t} \\ \omega_0, 3e^{r3,t} \\ \vdots \\ \omega_0, Me^{rM,t} \end{pmatrix} \tag{3.9}$$

Note that despite the fact that the notation *wt,k* is used to represent weights in Pth portfolio, they will not be normalized in this procedure, in such a way that $\sum_{k=1}^{N} w_{t,k}$ for $k \neq 0$ may be different than one.

5. The log-returns of the portfolio for each of the scenarios are estimated as:

$$R_t = \log(\sum_{k=1}^{N} w_{t,k}) \tag{3.10}$$

remembering that $\sum_{k=1}^{N} w_{t,k} = 1$

6. Categorize the portfolio returns (*Rt*) for the various scenarios into percentiles.
7. The VaR will be the return that correlate with the preferred confidence level. For instance, if there are 200 days and a confidence level of 99% preferred, the VaR will be the second worst return of the portfolio.

V. MONTE CARLO SIMULATION METHOD

Monte Carlo simulation is more pliable. Unlike historical simulation, Monte Carlo simulation permits the risk manager to use real historical distributions for risk factor returns as opposed to having to assume normal returns. Monte Carlo simulation is an extensive method of stochastic modeling processes-processes entailing human selection for which we have insufficient information. It imitates such a procedure by way of random numbers obtained from probability distributions which are presumed to correctly describe the un- known constituents of the process being modeled. Monte Carlo simulation is largely used in physics and engineering as well as in finance.

Stanislaw Ulam created the Monte Carlo approach in 1946 (Eckhardt, 1987) and includes some method of statistical sampling used to estimate solutions to quantitative problems. In the procedure, the arbitrary procedure under analysis is imitated time after time, where in each simulation will be generated a scenario of conceivable parameters of the portfolio at the target horizon. By creating a substantial number of plans, ultimately the distribution acquired through simulation will converge towards the true distribution. A good illustration of this method can be obtained, for example, in Holton (2003, chapter 5).

Crouhy et al. (2001) stated that this approach is beneficial in that: it allows the performance of sensitivity analyses and stress testing; the method can be used to model any complex portfolio; and that any distribution of the risk factors may be used. He however stated that outliers are not incorporated into the distribution; it is very computer intensive. In addition to the strength of Monte Carlo simulation is that no assumptions about normality of returns have to be made. The method is also capable of covering nonlinear instruments, such as options, Damodaran (2007). To add more to the benefits of this approach of VaR, Jorion (2001) reminds that Monte Carlo simulation initiates the whole distribution and consequently it can be used, for example, to estimate losses in excess of VaR. A possible weakness is also model risk, which arises due to wrong assumptions about the pricing models and underlying stochastic processes, a possible weak. If these are not properly stated, VaR calculations will be misrepresented, Jorion (2001). Furthermore, Dowd (1998) points out that complex techniques related to this approach necessitate specific skills. Senior management may therefore have difficult time acquainting themselves of how VaR values are calculated when Monte Carlo is used.

VI. COMPARISON OF METHODS

VaR methods vary in their propensity to capture risks of options, ease of execution, ease of interpretation to directors and managers, pliability in analyzing the effect of variations in the assumptions, and reliability of the results (Linsmeier and Pearson, 1996). As for the accuracy of the result, the best method seems to be the Monte Carlo method. The advantage exibility is particularly large. However, its use may be time consuming and it requires some knowledge and experience of the creators and users. Both Monte Carlo simulation and historical simulation methods rely on simulations and they suffer when using a lower number of scenarios by bad convergence to the actual sample quantile. While Monte Carlo method is generating larger number of scenarios, and the limits are given by the computational resources available, the historical simulation method exhibits a more serious problem a long time series are often not available and VaR cannot be estimated, especially at higher levels of probability. The optimal choice will be decided by which dimension the risk manager finds most significant and appropriate. If VaR is being calculated for a risk source that is stable and in the presence of real historical data, historical simulations provide good estimate. In the most comprehensive case of computing VaR for nonlinear portfolios over long time periods, where the historical data is volatile and non-stationary and the normality assumption is uncertain, Monte Carlo simulations do best (www.stern.nyu.edu/ adamodar/pdfiles/papers/VAR.pdf, 2010).

VII. A GENERAL SOLUTION TO THE BASIC VAR PROBLEM

Given that a portfolio consist of Assets 1, 2, 3, . . . , N .D_i Cedis are invested in Asset i, the total value of the portfolio is then then

$$D_1 + D_2 + D_3 + \ldots + D_N = \sum_{i=1}^{N} D_i = D \text{ Cedis.}$$

Assumption is made that the one-day Asset i return is normally distributed with variance $\sigma 2$ and expected value $E(ri)$. The covariance between the 1-day returns of Assets i and j is denoted by σij. We want to determine the 1-day VaR i at a confidence level of 5%

To solve this problem above, calculate the variance and expected return of the total port- folio. To do this, the weighting for each asset is calculated. The proportion of the portfolio expected return located to Asset i is

$$\psi_i = \frac{D_i}{D_1 + D_2 + D_3 + \ldots + D_N}$$

These are called asset weighting factors. Let $K = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \ldots \\ \psi_N \end{pmatrix}$ and $U = \begin{pmatrix} E|r_1| \\ E|r_2| \\ E|r_3| \\ \ldots \\ E|r_M| \end{pmatrix}$

A linear combination of random variables are created, where the random variables are the expected 1-day returns for each asset, and the coefficients are the asset weighing factors.

From the property of expectations,

$$E\left|\sum_{i=1}^{n} \alpha_i X_i\right| = \sum_{i=1}^{n} \alpha_i E|X_i|$$

and using the matrix method for finding this expectation: $E|r_{portfolio}| = E\left|\sum_{i=1}^{N} \alpha_i r_i\right| =$

$$K^T U = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \ldots \\ \alpha_N \end{pmatrix}^T \begin{pmatrix} E|r_1| \\ E|r_2| \\ E|r_3| \\ \ldots \\ E|r_M| \end{pmatrix}$$

$$E|r_{portfolio}| = \sum_{j=1}^{M} \psi_j E|r_i| = \mu_p$$

Subsequently, the variance of the total portfolio is calculated. The variance of the linear combination of random variables is given by:

$$Var(\sum_{i=1}^{n} \alpha_i X_i) = \sum_{i=1}^{n} \alpha_i^2 Var(X_i) + 2\sum\sum_{i\text{p} j}^{N} \psi_j \psi_i Cov(r_i, r_j)$$

The above equation is modified to the conditions of our stated problem:

$$\alpha_P^2 = Var(\sum_{i=1}^{N} \alpha_i r_i) = \sum_{i=1}^{N} alpha_i^2 Var(r_j) + 2\sum\sum_{i\text{p} j}^{N} \psi_j \psi_i Cov(r_i, r_j)$$

$$\alpha_p^2 = \sum_{i=1}^{N} \alpha_i^2 \alpha_j^2 + 2\sum\sum_{i\text{p} j}^{N} \alpha_j \alpha_i \alpha_{ij}$$

By computing α_p^2,

$$K = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \cdots \\ \alpha_N \end{pmatrix}^T \quad \text{and} \quad \sum = \begin{pmatrix} \alpha_1^2 & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_2^2 & \alpha_{23} & \cdots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_3^2 & \cdots & \alpha_{3M} \\ . & . & . & . & . \\ \alpha_{N1} & \alpha_{N2} & \alpha_{N3} & \cdots & \alpha_N^2 \end{pmatrix}$$

From the definition that:

$$Var(\sum_{j=1}^{n} \alpha_i X_i) = K^T \sum K$$

$$\Rightarrow \alpha_P^2 = Var(\sum_{j=1}^{n} \alpha_i r_i) = K^T \sum K$$

$$\alpha_P^2 = \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \cdots \\ \alpha_N \end{pmatrix}^T \begin{pmatrix} \alpha_1^2 & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_2^2 & \alpha_{23} & \cdots & \alpha_{2M} \\ \alpha_{31} & \alpha_{32} & \alpha_3^2 & \cdots & \alpha_{3M} \\ . & . & . & . & . \\ \alpha_{N1} & \alpha_{N2} & \alpha_{N3} & \cdots & \alpha_N^2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \cdots \\ \alpha_N \end{pmatrix}$$

The VaR can now be calculated since the expected value and variance for the overall port- folio return has been evaluated. Assume that the portfolio return is normally distributed with mean μp and variance α_P^2 both of which has already been calculated for. Since a 5% confidence level is needed, the return is solved such that a return worse than this return occurs only 5% of the time. Mathematically, we are solving for r*. Assume r* is found. Normally r* is minute, non-positive decimal. 100r*% is a percentage and can be thought of as the one-day percent loss such that, in normal market conditions, the portfolio loses more than 100 r*% only 5% of the time. Therefore, the one-day Value-at-Risk at a 5% confidence level is
$D\,|r*|$.

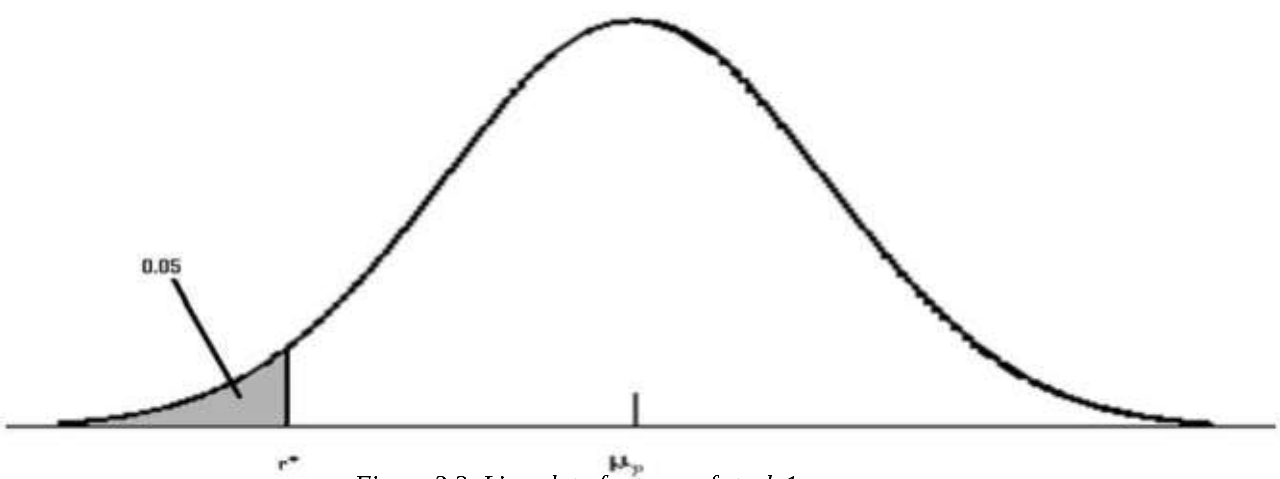

Figure 3.2: Line plot of returns of stock 1

In the rare occurrence that $r* > 0$, the VaR is not very beneficial. However, r* was evaluated such that the portfolio performs worse than r* only 5% of the time. But $r* > 0$, hence it can be stated that only 5% of the time will the portfolio earn us a positive return between 0 and r* or lose money. Therefore, if $r* > 0$ is obtained, it is an ineffective metric.
A new VaR analysis should then be estimated with a lower confidence level until we obtain an $r* < 0$.

Skewness: Skewness is a measure of the asymmetry of the data around the sample mean. If skewness is negative, the data are spread out more to the left of the mean than to the right. If skewness is positive, the data are spread out more to the right. The skewness of the normal distribution (or any perfectly symmetric distribution) is zero. This means normally distributed data is assumed to be symmetrically distributed around its mean. The skewness of a distribution is defined as

$$s = \frac{E(x-\mu)^3}{\alpha^3} \tag{3.11}$$

Where μ is the mean of x, σ is the standard deviation of x, and E(t) represents the expected value of the quantity t. Skewness computes a sample version of this historical value. Therefore a dataset with either a positive or negative skew deviates from the normal distribution assumptions. This can cause parametric approaches of VaR to be less effective if assets returns are heavily skewed, since these approaches assume that the returns are normally distributed.

Kurtosis: Kurtosis is a measure of how outlier-prone a distribution is. The kurtosis measures the peakedness of a data sample and describes how concentrated the returns are around their mean. A high value of kurtosis means more of the data's variance comes from extreme deviations.
The kurtosis of a distribution is defined as

$$k = \frac{E(x-\mu)^4}{\alpha^4} \tag{3.12}$$

Where μ is the mean of x, σ is the standard deviation of x, and E(t) represents the expected value of the quantity t.

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